TRENT UNIVERSITY
DEPARTMENT OF PHYSICS
PHYSICS 202H FINAL EXAMINATION

December 6, 2003 Time: 3 hours

PART A

Answer one (1) question.

1. Using the simple Bohr theory for circular orbits, calculate the energy required to remove the second electron from a once-ionized helium atom.

2. A particle of mass m is confined to a harmonic oscillator potential given by $V = mx^2\omega^2/2$, where $\omega^2 = K/m$ and K is the force constant. The particle is in a state described by the wave function

$$\Psi(x,t) = A\exp\left(\frac{-mx^2\omega}{2\hbar} - \frac{i\omega t}{2}\right).$$

Verify that this is a solution of Schrödinger's equation.

PART B

Answer three questions

3. In a Compton scattering event, the *scattered* photon was found to have an energy of 120 keV, and the recoil electron was given a kinetic energy of 40 keV. (a) What was the wavelength of the incident photon? (b) What was the scattering angle θ for the 120 keV photon? (c) What angle ϕ does the path of the scattered electron make with the direction of the incident photon?

4. Positronium is a hydrogen-like atom consisting of an electron and positron orbiting one another; the particles have identical masses, but opposite charges, each of magnitude e. Use the Bohr theory to obtain the allowed radii and energies. What is the wavelength of the photon emitted in the $n = 3 \to 2$ transition?

5. Given the following wave function for a wavepacket at time $t = 0$:

$$\Psi(x,t) = \psi(x) = \left(\frac{1}{\sqrt{\pi}\sigma}\right)^{1/2} \exp\left(ikx - \frac{x^2}{2\sigma^2}\right),$$

find the probability density, and calculate $\langle p_x \rangle$, and $\langle p_x^2 \rangle$.

6. A certain quantity x is uniformly distributed in the range $a \leq x \leq b$, *i.e.*, , the associated probability density is constant within that range, and zero everywhere else. Write down the normalized probability density function, and then show that

$$\sigma = \left(\langle x^2 \rangle - \langle x \rangle^2\right)^{1/2} = \frac{b-a}{\sqrt{12}}.$$

7. Show that when the recoil kinetic energy of the atom, $p^2/2M$, is taken into account, the frequency of a photon emitted in a transition between two atomic energy levels whose energy difference is ΔE is reduced by a factor which is approximately $(1 - \Delta E/2Mc^2)$. Evaluate for the case of the $n = 3 \to 2$ transition in hydrogen.

PART C

Answer one of the following questions.

8. A particle moves in a potential described by

$$V(x) = \begin{cases} 0, & -a/2 \le x \le a/2; \\ \infty, & \text{otherwise.} \end{cases}$$

The eigenfunctions are of the form $\psi_n(x) = A_n \cos(k_n x)$, or $\psi_n(x) = B_n \sin(k_n x)$, depending on the value of n.

(a) Write down an expression for $\psi_3(x)$.

(b) Normalize the corresponding wavefunction to unity.

(c) Evaluate $\langle x \rangle$, $\langle x^2 \rangle$, $\langle p \rangle$, $\langle p^2 \rangle$ in the $n = 3$ state. Then, evaluate the product $\Delta x \, \Delta p$, and verify that the result is consistent with the Heisenberg Uncertainty Principle.

9. Suppose that we want to evaluate $\langle xp \rangle$, *i.e.*, the expectation value of the product of position and momentum. Show that neither

$$\langle xp \rangle_1 = \int_{-\infty}^{\infty} \Psi^* x \left(-i\hbar \frac{\partial}{\partial x} \right) \Psi \, dx$$

nor

$$\langle xp \rangle_2 = \int_{-\infty}^{\infty} \Psi^* \left(-i\hbar \frac{\partial}{\partial x} \right) x \Psi \, dx$$

is acceptable because in both cases, the result is complex. Then show that

$$\langle xp \rangle = \int_{-\infty}^{\infty} \Psi^* \left[\frac{x \left(-i\hbar \frac{\partial}{\partial x} \right) + \left(-i\hbar \frac{\partial}{\partial x} \right) x}{2} \right] \Psi \, dx$$

yields an acceptable *i.e.*, real result. (Bear in mind that, in any realistic case, Ψ vanishes at $x = \pm\infty$.)

The following information may be useful:

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

$$\int x^2 \cos^2 bx \, dx = \frac{4\, b^3 x^3 + 3\,(2\, b^2 x^2 - 1) \sin 2bx + 6\, bx \cos 2bx}{24\, b^3}$$

$$\int_0^{\infty} \exp(-ax^2) \, dx = \frac{1}{2} \sqrt{\frac{\pi}{a}}$$

RECOMMENDED VALUES OF FUNDAMENTAL CONSTANTS

Quantity	Symbol	Value	Units
Speed of light in vacuum	c	299 792 458	$\mathrm{m\ s^{-1}}$
Newtonian constant of gravitation	G	6.673(10)	$10^{-11}\ \mathrm{m^3\ kg^{-1}\ s}$
Permeability of vacuum	μ_0	$4\pi \times 10^{-7}$	$\mathrm{N\ A^{-2}}$
Permittivity of vacuum	ϵ_0	$1/\mu_0 c^2$	
		$= 8.854\,187\,817\ldots$	$10^{-12}\ \mathrm{F\ m^{-1}}$
Planck constant	h	6.626 068 76(52)	$10^{-34}\ \mathrm{J\ s}$
		= 4.135 667 27(16)	$10^{-15}\ \mathrm{eV\ s}$
Planck constant $h/2\pi$	$\hbar$	1.054 571 596(82)	$10^{-34}\ \mathrm{J\ s}$
		= 6.582 118 89(26)	$10^{-16}\ \mathrm{eV\ s}$
Boltzmann constant	k	1.380 6503(24)	$10^{-23}\ \mathrm{J\ K^{-1}}$
		= 8.617 342(15)	$10^{-5}\ \mathrm{eV\ K^{-1}}$
Stefan-Boltzmann constant	σ	5.670 400(40)	$10^{-8}\ \mathrm{W\ m^{-2}\ K^{-}}$
Atomic mass unit	u	1.660 538 73(13)	$10^{-27}\ \mathrm{kg}$
		= 931.494 013(37)	MeV/c^2
Elementary charge	e	1.602 176 462(63)	$10^{-19}\ \mathrm{C}$
Fine structure constant	α	1/137.035 999 76(50)	
Electron mass	m_e	9.109 381 88(72)	$10^{-31}\ \mathrm{kg}$
		= 0.510 998 902(21)	MeV/c^2
Electron classical radius	r_e	2.817 940 285(31)	$10^{-15}\ \mathrm{m}$
Electron Compton wavelength	λ_C	2.426 310 215(18)	$10^{-12}\ \mathrm{m}$
Electron g-factor	g	−2.002 319 304 3737(82)	
Proton mass	m_p	1.672 621 58(13)	$10^{-27}\ \mathrm{kg}$
		= 938.271 998(38)	MeV/c^2
Proton-electron mass ratio	m_p/m_e	1836.152 6675(39)	
Neutron mass	m_n	1.674 927 16(13)	$10^{-27}\ \mathrm{kg}$
		= 939.565 330(38)	MeV/c^2
Neutron-electron mass ratio	m_n/m_e	1838.683 6550(40)	
Bohr radius	a_0	0.529 177 2083(19)	$10^{-10}\ \mathrm{m}$
Rydberg constant	R_∞	1.097 373 156 8549(83)	$10^{7}\ \mathrm{m^{-1}}$
Bohr magneton	μ_B	9.274 008 99(37)	$10^{-24}\ \mathrm{J\ T^{-1}}$
Nuclear magneton	μ_N	5.050 783 17(20)	$10^{-27}\ \mathrm{J\ T^{-1}}$
Avogadro's constant	N_A	6.022 141 99(47)	$10^{23}\ \mathrm{mol^{-1}}$

Useful Combinations of Constants

$$\frac{1}{4\pi\epsilon_0} = 8.987\,551\,796 \times 10^{9}\ \mathrm{N\ m^2\ C^{-2}}$$

$$\frac{e^2}{4\pi\epsilon_0} = 1.439\,965\,173\ \mathrm{eV\ nm}$$

$$hc = 1239.841\,86\ \mathrm{eV\ nm}$$