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Professor and Chair of the Department

W. Feng, BSc (Hebei), MSc (Shaanxi), PhD (Glasgow)

Professors

K. Abdella, BSc (Trent), MSc (Alberta), PhD (Western) (on leave 2018–2019)

M. Pollanen, BSc (Carleton), MSc, PhD (Toronto)

Associate Professor

S. Bilaniuk, BSc (Toronto), AM, PhD (Dartmouth)

Assistant Professor

W. Burr, BScEng, MSc, PhD (Queen's)

Adjunct Faculty

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The curriculum in Mathematics has been designed to accommodate a wide variety of interests in both pure and applied mathematics. All students in the Mathematics major program must take core courses in calculus, algebra, probability, and statistics. The intent is to provide students with both breadth and depth in Mathematics while allowing them to pursue those areas that interest them.

Notes

- MATH 1001H, 1080H, 2080Y, 2084H, and 2085H do not satisfy the Mathematics requirement for a Bachelor of Science degree.
- Students wishing to register for a reading course require either an 85% minimum in any prerequisite for the course or the chair's approval.

Bachelor of Science Program in Mathematics

- In addition to the program requirements listed below, students must satisfy the University degree requirements (see p. 15).
- The same course may not simultaneously satisfy the requirements of both programs in a joint-major degree.
- The following core courses are required for all Mathematics majors and joint-majors: MATH 1110H, 1120H, 1350H, 1550H, 2110H, 2200H, and 2560H.

The single-major Honours program. 20.0 credits including the following 11.0 credits:

- 5.0 MATH credits consisting of MATH 1110H and 1120H (or 1100Y or 1101Y), 1350H, 1550H, 2110H, 2120H, 2200H, 2350H, 2560H, and 4800H
- 0.5 credit from MATH 2150H, 2260H, 2600H, 3180H (or 2180H), or COIS 1020H
- 1.0 MATH credit from MATH 3150H, 3160H, 3310H, 3350H, 3510H, 3560H, 3610H, 3770H, or 3790H
- 1.0 MATH credit at the 2000 level or beyond in addition to the above
- 2.0 MATH credits at the 3000 level or beyond in addition to the above
- 1.5 MATH credits at the 4000 level in addition to the above
- 14.0 science credits are required for the Honours degree

The joint-major Honours program. 20.0 credits including the following 7.5 credits:

- 5.0 MATH credits consisting of MATH 1110H and 1120H (or 1100Y or 1101Y), 1350H, 1550H, 2110H, 2120H, 2200H, 2350H, 2560H, and 4800H
- 0.5 credit from MATH 2150H, 2260H, 2600H, 3180H (or 2180H), or COIS 1020H
- 1.0 MATH credit from MATH 3150H, 3160H, 3310H, 3350H, 3510H, 3560H, 3610H, 3770H, or 3790H
- 1.0 MATH credit at the 4000 level in addition to the above
- 14.0 science credits are required for the Honours degree

The single-major General program. 15.0 credits including the following 6.0 credits:

- 4.0 MATH credits consisting of MATH 1110H and 1120H (or 1100Y or 1101Y), 1350H, 1550H, 2110H, 2200H, 2350H, and 2560H
- 1.0 credit from MATH 2120H, 2150H, 2260H, 2600H, 3180H (or 2180H), or COIS 1020H
- 0.5 MATH credit from MATH 3150H, 3160H, 3310H, 3350H, 3510H, 3560H, 3610H, 3770H, or 3790H
- 0.5 MATH credit at the 3000 level or beyond in addition to the above
- 11.0 science credits are required for the General degree

The joint-major General program. 15.0 credits including the following 5.0 credits:

- 3.5 MATH credits consisting of MATH 1110H and 1120H (or 1100Y or 1101Y), 1350H, 1550H, 2110H, 2200H, and 2560H
- 0.5 credit from MATH 2120H, 2150H, 2260H, 2350H, 2600H, 3180H (or 2180H), or COIS 1020H
- 0.5 MATH credit from MATH 3150H, 3160H, 3310H, 3350H, 3510H, 3560H, 3610H, 3770H, or 3790H
- 0.5 MATH credit at the 3000 level or beyond in addition to the above
- 11.0 science credits are required for the General degree

The minor in Mathematics consists of 5.0 MATH credits, excluding MATH 1001H, 1005H, 1051H, 1052H (1050Y), 1080H, and 2080Y.

Specialization in Mathematical Finance

Students who have fulfilled the requirements for a single-major Honours degree in Mathematics may graduate with a Specialization in Mathematical Finance if they have successfully completed the following 6.5 credits:

- 5.0 MATH credits consisting of MATH 2150H, 2560H, 2570H (or 3570H), 3150H, 3160H, 3180H (or 2180H), 3350H, 3510H, 3560H, and 4510H
- 1.5 ECON credits consisting of ECON 1010H, 1020H, and 3020H

Note

- The following courses are recommended: COIS 1020H, MATH 4120H, MATH 4560H.

Specialization in Statistics

Students who have fulfilled the requirements for a single- or joint-major Honours degree in Mathematics may graduate with a Specialization in Statistics if they have successfully completed the following 4.5 credits:

- 3.0 MATH credits consisting of MATH 2560H, 2570H, 3180H, 3560H, 4560H, and 4561H
- 1.0 COIS credit consisting of COIS 1010H and 1020H
- 0.5 credit from MATH 4570H, 4850Y, 4851H, 4852H, or COIS 4400H

Note

- The following courses are strongly recommended for students planning to pursue graduate studies in statistics: MATH 2200H, 3160H, 3770H, 3790H, 4620H.

Please consult the academic timetable for information on courses that will be offered in 2018–2019, including when they will be scheduled.

» **MATH 1001H: Precalculus Mathematics (Sc)**

Designed for students needing to strengthen their math background in preparation for further courses in mathematics or the sciences. Topics include rational numbers, decimal representation, real numbers, inequalities, algebraic expressions; trigonometry; functions, including algebraic, exponential, logarithmic, and trigonometric functions; analytic geometry; simple sequences and series. Prerequisite: Ontario Grade 11 Functions and Relations (MCR3U) or equivalent. Not open to students enrolled in or with credit for any MATH courses other than MATH 1050Y, 1051H, 1052H, 1350H, or 2080Y. Does not satisfy the Mathematics requirement for a Bachelor of Science degree. Not for credit toward a major or minor in Mathematics.

» **MATH 1005H: Applied Calculus (Sc)**

An introduction to the methods and applications of calculus. Derivatives, exponential and logarithmic functions, optimization problems, related rates, integration, partial derivatives, differential equations. Selected applications from the natural and social sciences. Prerequisite: A Grade 12U mathematics course or its equivalent. Not open to students enrolled in or with credit for MATH 1100Y, 1101Y, 1110H, or 1120H. Not for credit toward a major or minor in Mathematics.

» **MATH 1051H: Non-Calculus Statistics I: Elementary Probability and Statistics (Sc)**

Data summary, elementary probability, discrete and continuous distributions, the central limit theorem, estimation and hypotheses testing. This course uses high school mathematics as a foundation and involves the use of computer software. Not open to students enrolled in or with credit for MATH 2560H. Not for credit toward a major or minor in Mathematics. Excludes MATH 1050Y.

» **MATH 1052H: Non-Calculus Statistics II: Elementary Statistical Methods (Sc)**

Linear regression models, goodness-of-fit and analysis of contingency tables, analysis of variance for completely randomized design, randomized block design and factorial design, nonparametric tests. Use of computer methods for analysis of real data sets. Prerequisite: MATH 1051H. Not for credit toward a major or minor in Mathematics. Excludes MATH 1050Y.

» **MATH-CCTH 1080H: Math for Everyday Life (Sc)**

Should you buy a ticket for 6/49 or Super 7? If you test positive for a rare disease, what is the chance that you actually have it? How are E. coli bacteria and interest rates related? These are some of the questions we investigate in this course. We also look into the use, misuse, and abuse of mathematics in the media. Prerequisite: Ontario Grade 11 Functions & Relations (MCR3U) or equivalent. Not open to students enrolled in or with credit for any other MATH courses. Does not satisfy the Mathematics requirement for a Bachelor of Science degree. Not for credit toward a major or minor in Mathematics.

» **MATH 1110H: Calculus I: Limits, Derivatives, and Integrals (Sc)**

Limits, differentiation, and integration through the basic substitution rule, with applications to other areas of mathematics and the physical and social sciences. Prerequisite: Grade 12 Advanced Functions or equivalent with at least 60%. Strongly recommended: Grade 12U Calculus and Vectors. Excludes MATH 1100Y, 1101Y.

» **MATH 1120H: Calculus II: Integrals and Series (Sc)**

Techniques and applications of integration, sequences, series, and power series, with applications to other areas of mathematics and the physical and social sciences. Prerequisite: MATH 1110H. Excludes MATH 1100Y, 1101Y.

» **MATH 1350H: Linear Algebra I: Matrix Algebra (Sc)**

Vectors, systems of linear equations, matrices, determinants, linear transformations, eigenvalues, and eigenvectors. Prerequisite: A Grade 12U mathematics course with at least 60%. Recommended: Grade 12U Calculus and Vectors.

» **MATH 1550H: Probability I: Introduction to Probability (Sc)**

Probability, random variables, probability distributions. Does not count as the Introductory Statistics course required for admission to some professional schools. Prerequisite: MATH 1005H or 1110H (or 1100Y or 1101Y) or permission of instructor.

» **MATH 2080Y: Mathematics for Teacher Education (Sc)**

A course in mathematics and mathematical thinking for prospective elementary school teachers. Number systems and counting, graphs and networks, symmetry and patterns, mathematics in nature and art, probability and statistics, measurement and growth. Excludes any MATH course, or its equivalent, which counts toward a major or minor in Mathematics. Does not satisfy the Mathematics requirement for a Bachelor of Science degree.

» **MATH 2110H: Calculus III: Calculus of Several Variables (Sc)**

Multivariable functions, curves, and surfaces in two and three dimensions. Partial differentiation and applications. Multiple integrals. Prerequisite: 60% or higher in MATH 1350H and in 1120H (or 1100Y or 1101Y).

» **MATH 2120H: Calculus IV: Vector Calculus (Sc)**

Parametric curves and surfaces, vector functions and fields. Line integrals, Green's Theorem. Surface integrals, curl and divergence, Stokes' and Divergence Theorems. Prerequisite: MATH 2110H.

» **MATH-PHYS 2150H: Ordinary Differential Equations (Sc)**

First order equations; qualitative and numerical methods. Second order linear equations. Linear systems. Applications to physical and biological models. Laplace transforms. Prerequisite: 60% or higher in MATH 1120H (or 1100Y or 1101Y). Recommended: MATH 1350H.

» **MATH 2200H: Mathematical Reasoning (Sc)**

Introduces concepts and methods that are essential for all advanced courses in pure mathematics. Intended for Mathematics majors early in their program. Logic, abstraction, proof techniques. Basic combinatorics. Sets, functions, (in/sur/bi)jections. Cantor's transfinite arithmetic. Number theory: divisibility, prime factorization, modular arithmetic. Prerequisite: MATH 1120H (or 1100Y or 1101Y) or 1350H.

- » **MATH 2260H: Geometry I: Euclidean Geometry (Sc)**
Elements of Euclidean geometry, stressing links to modern mathematical methods. Geometric transformations and symmetry. Recommended for Education students. Prerequisite: One of MATH 1005H, 1110H (or 1100Y or 1101Y), or 1350H.
- » **MATH 2350H: Linear Algebra II: Vector Spaces (Sc)**
Complex numbers, vector spaces, basis and dimension, linear transformations, diagonalization, quadratic forms, least squares, inner product spaces, orthogonality. Prerequisite: MATH 1110H (or 1100Y or 1101Y) and 1350H.
- » **MATH 2560H: Statistics I: Introduction and Applications (Sc)**
An introduction to applied statistical methods. Probability distributions and the central limit theorem, methods of point estimation, confidence intervals, hypothesis testing, comparative inferences, and nonparametric methods. Computer methods in statistics. Prerequisite: MATH 1550H and one of MATH 1005H or 1110H (or 1100Y or 1101Y).
- » **MATH 2570H: Probability II: Introduction to Stochastic Processes (Sc)**
Covers a variety of important models used in modelling of random events that evolve in time. These include Markov chains (both discrete and continuous); Poisson processes and queues. The rich diversity of applications of the subject is illustrated through varied examples. Prerequisite: MATH 1350H and 1550H. Excludes MATH 3570H.
- » **MATH-COIS 2600H: Discrete Structures (Sc)**
Mathematics related to computer science, including sets and relations, counting techniques and recursive relations, trees and networks. Applications to analysis of algorithms, data structure, and optimization problems. Prerequisite: MATH 1350H and 1120H (or 1100Y or 1101Y); or COIS 1020H, MATH 1350H, and one of MATH 1005H or 1110H.
- » **MATH-PHYS 3130H: Classical Mechanics (Sc) (see Physics & Astronomy)**
- » **MATH-PHYS 3150H: Partial Differential Equations (Sc)**
The heat equation, wave equation, telegraph equation, Laplace equation, Poisson equation and Schrodinger equation, in one, two, and three dimensions, in Cartesian, Polar, and/or Spherical coordinates. Physical interpretations. Properties of harmonic functions. Classification of PDEs. Fourier theory: uniform, pointwise, and L2 convergence. Initial/boundary value problems: existence and uniqueness of solutions. Solutions to I/BVPs using orthogonal eigenfunction expansions. Solutions to I/BVPs using convolutional transforms. Prerequisite: MATH 2110H and 2150H. Strongly recommended: MATH 1350H.
- » **MATH-PHYS 3160H: Methods of Applied Mathematics (Sc)**
Differential equations in applied mathematics, including Bessel, Legendre, hypergeometric, Laguerre, Hermite, Chebyshev, etc. Series and numerical solutions. Properties of the special functions arising from these equations. Prerequisite: MATH-PHYS 2150H. Recommended: MATH 2200H.
- » **MATH-COIS 3180H: Introduction to Numerical and Computational Methods (Sc)**
Error analysis, nonlinear equations, linear systems, interpolation methods, numerical differentiation and integration and initial value problems. Prerequisite: 60% or higher in MATH 1005H or 1110H (or 1100Y or 1101Y). Excludes MATH-COIS 2180H.
- » **MATH 3200H: Number Theory (Sc)**
Divisibility (GCDs, LCMs, Euclidean algorithm, Bezout's identity). Linear Diophantine Equations. Prime numbers (Factorization; Fermat/Mersenne numbers; pseudoprimes; Carmichael numbers). Modular Arithmetic (Chinese Remainder Theorem; Fermat/Euler theorem). Group of units mod m . Primitive roots. Quadratic Residues (Legendre symbols; Quadratic Reciprocity). Prerequisite: MATH 1350H and 2200H.

- » **MATH-COIS 3210H: Mathematical Cryptography (Sc)**
Public vs. private key cryptosystems: cyphertexts, plaintexts, and Kerkhoff's principle. Shannon's theory of perfect secrecy. Modular arithmetic: Chinese remainder theorem, Fermat/Euler theorems. RSA cryptosystem: definition and vulnerabilities. El-Gamal cryptosystem. Rabin cryptosystem. Quadratic residue theory. Probabilistic primality tests and factoring algorithms. Optional: discrete logarithm algorithms and elliptic curve cryptosystems. Prerequisite: MATH 2200H. Recommended: MATH-COIS 2600H or both MATH 1550H and COIS 2020H.
- » **MATH 3260H: Geometry II: Projective and Non-Euclidean Geometry (Sc)**
Elements of projective and non-Euclidean geometry, including an introduction to axiomatic systems. Prerequisite: MATH 1110H (or 1100Y or 1101Y) or 1350H.
- » **MATH 3310H: Algebra III: Introduction to Abstract Algebra (Sc)**
An introduction to the fundamental algebraic structures: groups, rings, fields. Subgroups and subrings, homomorphisms and isomorphisms, quotient structures, finite fields. Selected applications. Prerequisite: 60% or higher in MATH 2200H and 2350H. Excludes MATH 3320H, 3360H.
- » **MATH-COIS 3350H: Linear Programming (Sc)**
An introduction to the concepts, techniques, and applications of linear programming and discrete optimization. Topics include the simplex method, duality, game theory, and integer programming. Prerequisite: MATH 1350H.
- » **MATH 3510H: Mathematical Finance (Sc)**
Elements of stochastic calculus. Discrete time market models and continuous time market models. Self-financing strategies and arbitrage. Replication of claims. Completeness of market models. Pricing of derivatives: binomial model, Black-Scholes model. Historical and implied volatility. Prerequisite: MATH 1550H and 2150H.
- » **MATH 3560H: Statistics II: Linear Statistical Models (Sc)**
Linear and generalized linear models, introduction to smoothing and simulation, additive models, and topics in distributions and dimensionality. Prerequisite: MATH 1350H and 2560H. Recommended: MATH 2350H.
- » **MATH 3610H: Discrete Optimization (Sc)**
An introduction to the concepts, techniques, and applications of discrete optimization. Topics include integer programming, dynamic programming, network optimization, and approximation methods for NP hard problems. Prerequisite: MATH 1350H and one of MATH 2200H or MATH-COIS 2600H.
- » **MATH 3700H: Metric Geometry and Topology (Sc)**
Metric spaces. Limits and continuity. Completeness: the Baire Category Theorem; normed linear spaces and Banach spaces; the Contraction Mapping Theorem and applications. Compact, separable, and (first/second) countable spaces: the Heine-Borel and Lindelof theorems. Topological spaces. Hausdorff axiom and (non) metrizability. Product spaces and quotient spaces. Compactness and Tychonoff's theorem. (Path)-connectedness. Prerequisite: MATH 1120H (or 1100Y or 1101Y) and 2200H.
- » **MATH 3770H: Complex Analysis (Sc)**
Complex numbers, limits and series. Complex functions, holomorphic functions. Complex contour integrals; Cauchy integral theorems. Taylor series. Laurent series. Calculus of residues. The Fundamental Theorem of Algebra. Frequency analysis: Fourier transform, Laplace transform, Fourier series, Z-transform. Prerequisite: MATH 2120H.
- » **MATH 3790H: Real Analysis (Sc)**
The real number system. Limits. Continuity. Differentiability. Mean-value theorem. Convergence of sequences and series. Uniform convergence. Prerequisite: Prerequisite: 60% or higher in MATH 1120H (or 1100Y or 1101Y) and 2200H.

- » **MATH 3810H: Ancient and Classical Mathematics (Sc)**
Traces the historical development of mathematics from prehistory to medieval times, and the interactions between the development of mathematics and other major trends in human culture and civilization. We study the mathematics of ancient Egypt and Mesopotamia, and classical Greece and Rome. Prerequisite: MATH 1120H (or 1100Y or 1101Y). Recommended: MATH 2200H or 2350H.
- » **MATH 3820H: Mathematics from Medieval to Modern Times (Sc)**
Traces the development of mathematical ideas, abstraction, and proofs. The genesis of modern arithmetic in medieval India, the birth of algebra in the Islamic world, and their influence on medieval European mathematics. Renaissance mathematics (polynomial equations, analytic geometry). The Enlightenment (calculus, number theory). The apotheosis of rigour since the nineteenth century. Prerequisite: MATH 1120H (or 1100Y or 1101Y). Recommended: MATH 2200H or 2350H.
- » **MATH 3900Y, 3901H, 3902H, 3903H, 3904H: Reading Course (Sc)**
Details may be obtained by consulting the department. Prerequisite: 85% minimum in any prerequisite for the course; or permission of department chair.
- » **MATH 4120H: Mathematical Modelling I (Sc)**
Provides an introduction to the mathematical modelling process and applies this process to simple mathematical modelling problems arising from a variety of application areas in science and engineering. Mathematical modelling techniques, such as differential equations, discrete systems, and numerical methods along with computer aids are utilized. Prerequisite: MATH-PHYS 2150H.
- » **MATH-PHYS 4140H: Advanced Classical Mechanics (Sc) (see Physics & Astronomy)**
- » **MATH 4180H: Advanced Numerical Methods (Sc)**
Deals with a variety of numerical methods for solving ordinary and partial differential equations arising from scientific and engineering applications. Topics include finite difference, adaptive techniques, multi-step methods, Runge-Kutta methods, direct and iterative methods for systems, stability and convergence. Prerequisite: MATH 2150H and 3180H (or 2180H).
- » **MATH-COIS 4215H: Mathematical Logic (Sc)**
An introduction to the syntax and semantics of propositional and first-order logics through the Soundness, Completeness, and Compactness Theorems. Prerequisite: MATH 2200H.
- » **MATH-ECON 4251H: Advanced Mathematical Methods for Economics (Sc)**
Provides the fundamental mathematical methods that are required to understand advanced topics in economics and to analyze economic models. Also useful for students who intend to pursue graduate studies in economics, business, or applied mathematics. Topics include differential equations, difference equations, optimization, optimal control theory, and calculus of variation. Prerequisite: ECON 3250H or both MATH 2110H and 2150H.
- » **MATH 4310H: Algebra IV: Galois Theory (Sc)**
Extension fields and Galois groups; the fundamental theorem of Galois Theory; the insolubility of the quintic. Prerequisite: MATH 3310H.
- » **MATH 4400H: Mathematics Through Problem-Solving (Sc)**
A seminar-based course in problem solving. Topics include invariants, number properties, polynomials, functional equations, inequalities, combinatorial games. Prerequisite: MATH 2200H. Excludes MATH 4951H.
- » **MATH 4510H: Mathematical Risk Management (Sc)**
Basic mathematical theory and computational techniques for how financial institutions can quantify and manage risks in portfolios of assets. Topics include: mean-variance portfolio analysis, the capital asset pricing model and Value at Risk (VaR). Prerequisite: MATH 1550H and 2110H.

» **MATH 4560H: Topics in Statistics (Sc)**

Selected topics in statistics, including time series analysis, advanced modeling and regression, computational statistics, and computational algorithms for statistics. Prerequisite: MATH 2560H and one of MATH 3560H or 2570H (or 3570H). Recommended: MATH 2350H.

» **MATH 4561H: Sampling and Design of Experiments (Sc)**

Sampling and design of experiments. Topics in design of experiments include ANOVA, randomized block designs, factorial designs, blocking and confounding in factorial designs, response surface methods. Topics in sampling include simple random, systematic, stratified and cluster sampling, sample size estimation, unequal probability sampling, and multistage designs. Prerequisite: MATH 2350H and 2560H. Recommended: MATH 3560H.

» **MATH 4570H: Topics in Probability: A Second Course in Stochastic Processes (Sc)**

Continuous-time Markov chains, renewal and queueing theory, Brownian motion, and an introduction to simulation of processes. Extensions include stochastic calculus and stochastic differential equations. Prerequisite: MATH 2570H (or 3570H). Recommended: MATH 2560H and 3790H.

» **MATH 4610H: Introduction to Graph Theory (Sc)**

An introduction to graph theory with emphasis on both theory and applications and algorithms related to computer science, operation research, and management science. Prerequisite: MATH 2200H or MATH-COIS 2600H.

» **MATH 4620H: Introduction to Combinatorics (Sc)**

An introduction to combinatorics. The topics include counting techniques, generating functions, and block design. Prerequisite: MATH 2200H.

» **MATH 4710H: Chaos, Symbolic Dynamics, Fractals (Sc)**

An introduction to discrete dynamical systems. Periodicity, attraction. Parametrized families of functions, bifurcation, chaos. Symbolic dynamics, conjugacy, Cantor Sets. Deterministic fractals, fractal dimension, Lyapunov exponents, entropy. Prerequisite: MATH 3700H or 3770H or 3790H.

» **MATH 4800H: Mathematics Honours Project (Sc)**

Working under the guidance of an approved faculty supervisor, students independently study an area of mathematics, write a paper on the topic, and give a presentation describing the research conducted. Prerequisite: Any two of MATH 3150H, 3160H, 3310H, 3350H, 3510H, 3560H, 2570H (or 3570H), 3610H, 3770H, or 3790H.

» **MATH 4810H: Perspectives in Mathematics I (Sc)**

Team-taught by three instructors. Each instructor teaches a four-week module on a special topic. Prerequisite: 60% or higher in 1.0 3000- or 4000-level MATH credit.

» **MATH 4850Y, 4851H, 4852H: Community-Based Research Project (Sc)**

Students are placed in research projects with community organizations in the Peterborough area. Each placement is supervised jointly by a faculty member and a representative of a community organization. For details see Community-Based Research Program ([p. 406](#)). Prerequisite: MATH 2560H, one of MATH 3560H or 4561H, and a cumulative average of at least 75%. Students may take either MATH 4850Y or, one or both of 4851H and 4852H for credit.

» **MATH 4900Y, 4903H, 4904H: Reading Course (Sc)**

Details may be obtained by consulting the department. Prerequisite: 85% minimum in any prerequisite for the course, or permission of department chair.