Trent University
Department of Physics - PHY380H

Saturday, April 12, 2003

Final Exam

3 Hours

Questions:

Q1 (compulsory). (10pts) Consider a wave from a coherent source incident on a screen with small linear opening of length a. At a point P to the right of the screen, the contribution to the diffraction of a single secondary source from this opening is of the form: $\psi_s(r,t) = Re\left\{A\frac{e^{i(\omega t - kr)}}{r}\right\}$

(a) What kind of wave does it represent and what is its shape
(b) what represent each term in this expression
(c) what is the total contribution of the linear distribution of coherent sources
(d) what is the condition for the Fraunhoffer approximation

Answer 3 of the 6 above questions:

Q2. Explain how the impedance presented by a medium to electromagnetic (EM) waves is analogous to the impedance presented by a medium to acoustic waves. Are there any energy considerations to support your arguments?

Q3. The Pointing vector is defined as: $\vec{S} = \vec{E} \times \vec{H}$

(a) How is it related to the EM energy?
(b) How is it related to wave propagation?
(c) How is it related to the oscillations of the vector **E** and the vector **H**?

Q4. What are the differences between spherical and plane electromagnetic waves?

Q5. Why either the electrical field E-vector or the field vector-H is sufficient to describe the EM waves?

Q6. Comment on the above statements

(a) The electrical field $\vec{E}$ is in some cases perpendicular to the field-vector $\vec{H}$.

(b) The x-components of $\vec{E}$ and $\vec{H}$ are proportional

(c) The y-components of $\vec{E}$ and $\vec{H}$ are proportional

Q7. Comment on the above statements

(a) The electrical field $\vec{E}$ is perpendicular to the x-direction all the time

(b) The electrical field $\vec{E}$ is perpendicular to the direction of propagation all the time

<u>Problem1</u> (compulsory): (20pts)

A damped system consists of a mass m, on a spring of spring constant k, with a viscous damping proportional to the velocity of the mass (the proportionality or damping factor is b). It is described by a function *y(t)*. The natural frequency of the system is $\omega_0 = \sqrt{\frac{k}{m}}$.

1. Write the differential equation which describes y(t) (2pts)
2. If the system is critically damped, what is the relation between M, k and b? (3pts)
3. If the system is overdamped, what is the relation between M, k and b? (3pts)
4. If the system is overdamped, the motion may be described by:

$$y(t) = A\exp(-t/\tau_1) + B\exp(-t/\tau_2)$$

(a) Give the expressions of τ_1 and τ_2. Which is the fast term and which is the slow term? Explain (4pts)

(b) Show that the two terms fit the two relations:

- $\tau_1.\tau_2 = m/k$ (2pts) and $\tau_1 + \tau_2 = b/k$ (2pts)

5. Plot y(t) for two different damping factors b1 and b2 (b1>b2) **(4pts)**

Answer 3 of the above 4 problems

Problem2: (10pts)

Verify that $y(t,x) = Re\{A\exp i(\omega t - kx)\}$ is a solution of the one-dimensional wave equation. What kind of wave is this? Plot $y(x,t)$ for a given position and then for a given time. (5pts)

Verify that the velocity of a particle of the medium where the wave is traveling, is proportional to the slope of the function $y(x,t)$. (5pts)

Problem3: (10pts)

A string of infinite length is initially straight. At t= 0, it is given an initial "triangular" velocity profile described by the function:

$$\psi(x) = \begin{cases} 0 & x < -a \\ v_0(1 + x/a), & -a \le x \le 0 \\ v_0(1 - x/a), & 0 \le x \le a \\ 0 & x > a \end{cases}$$

(a) What is the general function y(x,t) which describes the string waveform? (2pts)

(b) What is the function $\varphi(x)$ which describes the initial string shape? (2pts)

(c) Find the unique solution to the wave equation subject to the given initial conditions. (6pts)

Problem4:

Consider a tube of semi infinite extent containing a fluid medium. A piston is set at the end x=0 of the tube.

(a) Write the wave equation describing the wave motion and give the solution for the fluid displacement assuming that there is no reflecting surface in x>0. (2pts)

At t<0, the fluid is in the undisturbed state and for t≥ 0 the piston is displaced from its initial position and given a displacement described by the function B(t).

(b) Give the boundary condition (valid at any time t) knowing that the particles adjacent to the piston are permanently labeled with the parameter x= 0. (2pts)

(c) find the wave function if the piston moves with constant velocity v for $t \geq 0$ i.e.

$$B(t)=\begin{cases}0 & t<0\\ vt & t\geq 0\end{cases} \quad \text{(3pts)}$$

(d) show that the excess pressure at any point and any time is given by:

$$p(x,t)=\begin{cases}0 & x>ct\\ v\sqrt{\rho_0\beta_a} & x<ct\end{cases} \quad \text{(3pts)}$$

where ρ_0 is the density of the fluid and β_a is its adiabatic bulk modulus.

Probleme5: (10pts) in this problem we assume that air is an ideal gas

(a) As the acoustic wave propagates through air, will the compressions and expansions of the gas take place isothermally or adiabatically? How do you decide?

(b) Starting with the equation of state of an ideal gas show that the velocity of sound in air is proportional to $\sqrt{T}$.

(c) What is the velocity of sound in air at 0^0C and at 20^0C

(d) Compare these velocities with the average velocities of the molecules resulting from the kinetic energy due to the temperature of the air.

The thermal energy is the average translational kinetic energy possessed by free particles and is given by the equipartition theorem:

$$\overline{E}_K = \frac{3}{2}k_B T$$

T is the absolute temperature, k_B= 1.38066 10^{-23} J/K the Boltzman constant, and R=8.314 J/mol.K the gas constant

for air: the adiabatic constant γ= 1.4, the molecular mass M= 28.95 g/mol.

Good luck