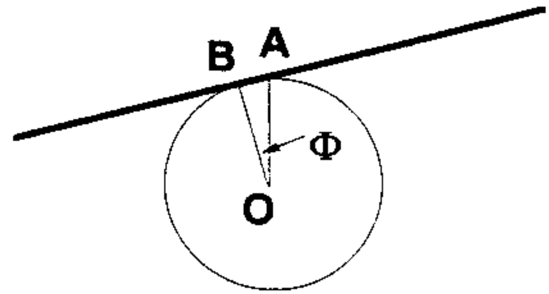
Trent University
Department of Physics
Physics 380H, Waves
Final Examination
Wednesday, April 18, 2001

Best 5 of questions 1-6 are worth 15% each; question 7 is worth 25%

1. A thin rod of mass, M (kg), and length, L (m), rocks about its center, A, on a cylinder of radius, R (m), as shown. When the angle with the horizontal is Φ, the contact point on the cylinder is B, a distance $R\Phi$ along the rod from A. Show that small displacements (i.e. small Φ) give rise to simple harmonic motion with a period given by $T^2 = \pi^2 L^2 / 3gR$ (s²). (hint: the moment of inertia of a thin rod about any point near its centre of mass may be expressed as $ML^2/12$ (kg.m²))



2. Two very long, horizontal wires having different linear densities, ρ_1 and ρ_2 (kg/m), are joined at $x = 0$. Tension, T (N), is applied to the entire system. An incident transverse wave given by, $y_1(x,t) = Y_1 \exp[i(\omega t - k_1 x)]$, travels to the right along the wire in the left region, $x < 0$. It is partly reflected and partly transmitted at $x = 0$. Find the reflected and transmitted amplitudes in terms of Y_1 and the ρ's on either side of $x = 0$. (Be sure to state your boundary conditions clearly.) What is the wavelength in the right wire, λ_2, given in terms of λ_1? Verify that your answers are valid when $\rho_1 = \rho_2$.

3. The speed of sound waves in a medium is a function of the volume density of the medium, ρ (kg.m³), and the elasticity of the medium, β (Pa). [The latter is called the bulk modulus, and is a measure of the pressure change required for a fractional change of volume, $-\Delta p/(\Delta V/V)$] The speed has been defined for various situations in this course by: $c^2 = \beta/\rho_0$, $c^2 = \gamma P_0/\rho_0$ and by $c^2 = \gamma R T_0/M$

3a) Comment on the relative speeds of sound, c_A and c_B, for each of the following four different situations: (in each case, the samples are called A and B)

i) Two samples of a rubber material, each of the same density. A is very rigid and B is very pliable.

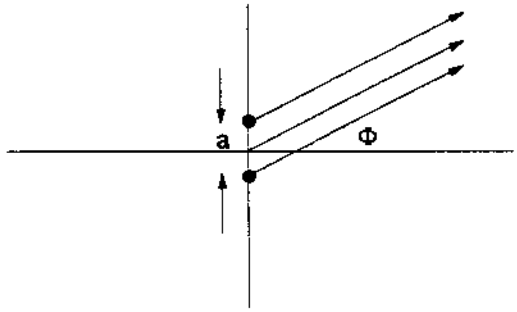
ii) A and B are gases at the same temperature. A's molecular weight is 2x B's.

iii) Two samples of a similar gas. A is hotter than B.

iv) A is a copper pipe and B is a steel pipe.

3b) The pitch of a note (i.e. its frequency) coming from a musical wind instrument such as a trumpet will change as the instrument's temperature rises. A musician calls this "warming-up"; she can re-tune her instrument by adjusting its length. Explain whether it should be lengthened or shortened when an instrument warms-up.

4. Two sources of periodic waves are situated at $y = +a/2$ and $y = -a/2$ along the $x=0$ plane. They are spaced by $a = 2\lambda$ apart. The waves they produce have common amplitudes, A, frequency, ω, and wave-number, k. The phase difference between the emitted waves is π radians at the sources. At a distant point, P, (defined by the angle Φ with respect to the right-bisector of the sources) the waves interfere.

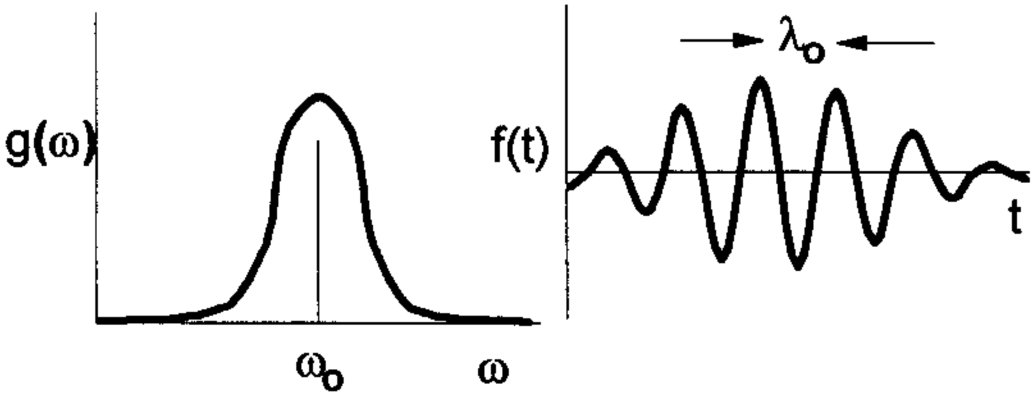


a) Derive an expression for the polar pattern, $I(\Phi)$, for the intensity of the waves at P?
b) Sketch the polar graph, indicating the directions of any maxima or minima.
c) In words, state how the pattern would change if the wavenumber, k, is very slightly decreased while the spacing, a, remains constant. (derivation not required.)

5. A stereo sound system is played at one end in a long room. The room has a cross section of about 3.3m x 3m (say 10 m^2). A sound-level meter near the middle of the room indicates that the sound level is at 120 db. (The reference standard of the db system for sound, I_0, is a power flux level of 10^{-12} W/m^2 .)
a) What approximately is the power flux, I, of the loud sound in the room in W/m^2 ?
b) What approximately is the total power, P, leaving the stereo system?

6. a) Fourier analysis defines the transformation of any single-valued f(t) to a complimentary, unique fourier transform, $g(\omega)$. In words, describe how a distribution of "cos" and "sin" functions with different ω's can combine to generate any f(t).

b) A Gausian wave packet is a set of waves which have frequencies that are defined by a "Gausian distribution" about some central frequency, ω_0. The Gausian frequency distribution, $g(\omega)$, is shown in the following figure. The Fourier transform of this function, f(t) is show in the figure on the next page. (Photon pulses are often drawn this way.) The function f(t) is a $\cos(\omega_o t)$ function contained within another Gausian shaped envelope which falls off to zero symmetrically on either side of a maximum. The mathematical functions (not given or required here) show that the widths of the Gausian parts of $g(\omega)$ and f(t) are related: if the $g(\omega)$ function is narrow, then the envelope of the f(t) function is wide; conversely, if the g(ω) function is broad, then the envelope of the f(t) function is narrow. Using either words or diagrams, (but without any maths), explain this example. Think of the limiting cases of g(ω) being extremely broad or extremely narrow. Note that λ_0 which is part of f(t) is related to ω_0 which is part of g(ω).



7) Answer each of the following short questions: (2% each)

a) What are some common features of all waves?
b) Compare "the wave equation" and "the simple harmonic motion equation"
c) What is meant by "damping" when we study waves?
d) Comment on why we preferred to use Aexp[i(ωt-kx)] rather than Acos(ωt-kx).
e) What is the approximate range of frequencies that humans can hear with ears?
f) What is the approximate range of frequencies that humans can see with eyes?
g) What is the approximate speed of sound in air as a % of the speed of light?
h) If an organ pipe of length 1 meter is open at both ends, what is its frequency?
i) If CHEX radio has a frequency of 980 Khz, what is its wavelength?
j) If an infrared scanner detects λ of 1.3 microns, what is the frequency?
k) Is it sufficient to know a guitar string's length, L, to determine its pitch?
l) In Young's experiment what is the relationship between d and λ ?
m) What did Michelson hope to observe by his interferometer experiment?
n) Why does an isolated loud speaker sound so poor compared to its operation in a closed "speaker box"?