Physics 380H - Wave Theory | **Homework #07 - Solutions**
Fall 2004 | Due 12:01 PM, Monday 2004/11/08

[50 points total]

"Journal" questions:
– What physics material do you recall from your elementary school experiences (up to about age 12)? How was it presented? What was your perception of the instructor's attitude toward the material? What about other non-physics sciences? Math?
– Any comments about this week's activities? Course content? Assignment? Lab?

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1. What experimental evidence of the wavelike nature of light led to the abandonment of the then accepted corpuscular theory by Poisson? What effect revived a modified corpuscular theory in the early 20th century due to a new theoretical framework of electromagnetic waves? Limit your discussion to about 50 words or so. [10]

   Solution: According to Towne in Section 11-7, Poisson asked Arago to demonstrate that the shadow of a circular disc noes *not* have a bright spot at the centre, which is one of the consequences of Fresnel's theory (based on the wave ideas of Young and Huygens which did not have widespread acceptance). Surprisingly, Arago's results were in exact agreement with the wave theory - there was a bright spot at the centre. In the early 20th century, a Einstein's theoretical model of the "Photoelectric Effect" gave rise to the idea of "photons", within the framework of quantum theory. Thus today we think of light as having a dual nature, both particle-like/corpuscular and wave-like.

2. (From Towne P11-11, pg 252)A yacht club Y is located on the semicircular shoreline of a harbour protected by a breakwater BW (see Towne, Figure 11-32). it is found that the most damaging storms are associated with swells which come from due north and have a wavelength of 20 ft. To give the yacht club maximum protection from these storms, it is decided to have two openings in the breakwater at C and C' equidistant from the centre O. (Consistent with the fictitious nature of the problem, simplifications should be made as desired.)

   (a) What should the spacing be betwen the openings? [10]

   Solution: If we assume that the size of the harbour is large enough so that we can use the Fraunhofer approximation, we can find the intensity of the waves as a function of the angle (measured from the centre of the arc of the harbour) via

$$I(\theta) = 4I_0 \cos^2 \left[\frac{(\Delta\phi)_0}{2} + \frac{kd}{2} \sin\theta \right]$$

   where $k = 2\pi/\lambda$. Since the waves hit the barrier head-on, there is no phase difference between the waves coming through the two gaps, so $(\Delta\phi)_0 = 0$. We want the intensity at the given angle $\theta_Y = 20^\circ = \pi/9\,\text{rad}$ to be a minimum, so

$$\begin{aligned}
I(\theta_Y) = 0 \quad \Longrightarrow \quad \cos^2\left[\frac{kd}{2}\sin\theta_Y\right] &= 0 \\
\frac{kd}{2}\sin\theta_Y &= \left(n+\frac{1}{2}\right)\pi, \qquad n=0 \\
\frac{kd}{2}\sin\theta_Y &= \frac{\pi}{2} \\
d &= \frac{\pi}{k\sin\theta_Y} = \frac{\lambda}{2\sin\theta_Y} \\
d &= \frac{(20\,\text{ft})}{2(0.3420\ldots)} = (29.2380\ldots\ \text{ft}) \\
d &\approx 29\,\text{ft}.
\end{aligned}$$

Thus the gaps in the breakwater should be about 29 ft apart in order to provide maximum protection for the club.

(b) Along how much of the shoreline on either side of Y will the wave amplitude during a storm be less than half of what it would be with only a single opening? (Take the radius $OY = 1500\,\text{ft}$. [10]

Solution: For this problem, I am unable to get Towne's result of 270 ft. Calculations based on intensity give about 250 ft, while calculations based on wave amplitude give about 160 ft of shoreline.

Given the spacing d calculated above, we would like to find the angles $\theta_{1/2}$ close to $\theta_Y = 20^\circ = \pi/9\,\text{rad}$ where the intensity is $I_0/2$.

$$\begin{aligned}
I(\theta_{1/2}) = \frac{I_0}{2} &= 4I_0\cos^2\left[\frac{kd}{2}\sin\theta_{1/2}\right] \\
\frac{1}{8} &= \cos^2\left[\frac{kd}{2}\sin\theta_{1/2}\right] \\
\cos\left[\frac{kd}{2}\sin\theta_{1/2}\right] &= \pm\frac{1}{2\sqrt{2}} = \pm(0.353553\ldots) \\
\frac{kd}{2}\sin\theta_{1/2} &= \cos^{-1}\left[\pm\frac{1}{2\sqrt{2}}\right] = (1.2094\ldots)\ \text{or}\ (1.9321\ldots)
\end{aligned}$$

If we let $c_1 = 1.2094\ldots$ and $c_2 = 1.9321\ldots$, then we can calculate the arc length between the two angles via $s = R\Delta\theta$ via

$$\begin{aligned}
\sin\theta_1 &= c_1\frac{2}{kd} \\
\theta_1 &= \sin^{-1}\left(\frac{\lambda c_1}{\pi d}\right) \\
\theta_1 &= \sin^{-1}\left(\frac{(20\,\text{ft})(1.2094\ldots)}{\pi(29.2380\ldots\ \text{ft})}\right) \\
&= \sin^{-1}(0.26333\ldots) \\
&= 0.26647\ldots\ \text{rad} = 15.26\ldots^\circ
\end{aligned}$$

$$\begin{aligned}
\sin\theta_2 &= c_2\frac{2}{kd} \\
\theta_2 &= \sin^{-1}\left(\frac{\lambda c_2}{\pi d}\right) \\
\theta_2 &= \sin^{-1}\left(\frac{(20\,\text{ft})(1.9321\ldots)}{\pi(29.2380\ldots\ \text{ft})}\right) \\
&= \sin^{-1}(0.42070\ldots) \\
&= 0.43422\ldots\ \text{rad} = 24.87\ldots^\circ
\end{aligned}$$

$$\begin{aligned}
\Delta\theta = \theta_2 - \theta_1 &= 0.16774\ldots\ \text{rad} = 9.61\ldots^\circ \\
s = R\Delta\theta &= (1500\,\text{ft})(0.16774\ldots) = 251.6\ldots\ \text{ft} \\
s &\approx 250\,\text{ft}
\end{aligned}$$

Thus there is about 250 ft of shore including the yatch club where the wave intensity will be less than half of what it would be with only a single opening.

If we look at the wave amplitude rather than the intensity we have that the following expression holds, where the $\cos(\omega t - \bar{\phi})$ factor does not influence the wave amplitude,

$$\psi(\theta) = 2A\cos\left[\frac{\Delta\phi}{2}\right]\cos(\omega t - \bar{\phi})$$

The phase difference between the two sources $\Delta\phi$ in the limit of large distances R approximation is given by $\Delta\phi = kd\sin\theta$. Since we are interested in only the magnitude of the amplitude, and not the sign (which is just another phase factor) we want to find the angles where

$$\begin{aligned}
\frac{A}{2} &= 2A\cos\left[\frac{kd}{2}\sin\theta_{1/2}\right] \\
\frac{1}{4} &= \cos\left[\frac{kd}{2}\sin\theta_{1/2}\right] \\
\frac{kd}{2}\sin\theta_{1/2} &= \cos^{-1}\left(\pm\frac{1}{4}\right) = (1.3181\ldots) \text{ or } (1.8234\ldots)
\end{aligned}$$

If we let $c_1 = 1.3181\ldots$ and $c_2 = 1.8234\ldots$, then we can calculate the arc length between the two angles via $s = R\Delta\theta$ via

$$\begin{aligned}
\sin\theta_1 &= c_1\frac{2}{kd} \\
\theta_1 &= \sin^{-1}\left(\frac{\lambda c_1}{\pi d}\right) \\
\theta_1 &= \sin^{-1}\left(\frac{(20\,\text{ft})(1.3181\ldots)}{\pi(29.2380\ldots\ \text{ft})}\right) \\
&= \sin^{-1}(0.28700\ldots) \\
&= 0.29109\ldots\ \text{rad} = 16.678\ldots^\circ
\end{aligned}$$

$$\begin{aligned}
\sin\theta_2 &= c_2\frac{2}{kd} \\
\theta_2 &= \sin^{-1}\left(\frac{\lambda c_2}{\pi d}\right) \\
\theta_2 &= \sin^{-1}\left(\frac{(20\,\text{ft})(1.8234\ldots)}{\pi(29.2380\ldots\ \text{ft})}\right) \\
&= \sin^{-1}(0.42070\ldots) \\
&= 0.3970\ldots\ \text{rad} = 22.74\ldots^\circ
\end{aligned}$$

$$\begin{aligned}
\Delta\theta &= \theta_2 - \theta_1 = 0.1059\ldots\ \text{rad} = 6.07\ldots^\circ \\
s &= R\Delta\theta = (1500\,\text{ft})(0.1059\ldots) = 158.9\ldots\ \text{ft} \\
s &\approx 160\,\text{ft}
\end{aligned}$$

Thus for about 160 ft of shore including the yatch club will have wave amplitudes less than half of what they would be with only a single opening.

3. (From Towne P11-27, pg 256) It is desired to exhibit localized interference fringes in sodium yellow light by forming a thin air wedge between two optical flats. The flats are 5 cm long and the wedge is formed by inserting a think piece of paper between the flats along one edge. How thick must the paper be if the fringes are to be 1 mm wide? Hint: you will need to determine the optical path difference between light reflected off of the top of the triangular air wedge and off of the bottom of the air wedge, and how that path difference is a function of the distance from the corner, where the two flats come into contact. [10]

   Solution: The wavelength of the soidum yellow light is about $\lambda = 589\,\text{nm}$. The change in optical path length difference $\Delta\delta$ between adjacent fringes will be a single wavelength of the light, so $\Delta\delta = \lambda$, and this will be equal to twice the change in height between these two points, so $\Delta\delta = \lambda = 2\Delta h$. We also know that the distance between the two fringes is $l = 1\,\text{mm} = 0.1\,\text{cm}$. Since the total length of the plate is $L = 5\,\text{cm}$, we will have similar triangles formed for each fringe step compared to the structure as a whole:

$$\begin{aligned} \frac{\Delta h}{l} &= \frac{t}{L} \\ t &= L\frac{\Delta h}{l} = \frac{L\lambda}{2l} \\ &= \frac{(5\,\text{cm})(589\,\text{nm})}{2(0.1\,\text{cm})} = 14725\,\text{nm} \approx 15\,\mu\text{m} \end{aligned}$$

   The edge of the wedge and thus the paper must be about $15\,\mu\text{m}$ in thickness.

4. When tuning a piano, “Middle A” should be at 440 Hz. Using a Middle A tuning fork you determine that the piano that you are working on is a little bit “sharp” and when Middle A is played, you hear beat interference between the piano and the tuning fork of about 10 cycles in about 7 seconds. What is the frequency that the piano is tuned to? [10]

   Solution: The beat frequency is merely the difference in the frequency of the two sources (note that the beat frequency is twice the frequency of the sinusoidal function governing the amplitude of the resultant wave function, since squaring this wave function to get an intensity gives an intensity with periodicity that is twice the periodicity of the wave function amplitude). If $f_1 = 440\,\text{Hz}$ is the tuning fork frequency, and $f_b = 10/7\,\text{Hz}$, then

$$f_b = |f_2 - f_1| \quad \Longrightarrow \quad f_2 = 440\,\text{Hz} + \frac{10}{7}\,\text{Hz} = 441.42857\ldots\,\text{Hz} \approx 441.43\,\text{Hz}.$$

   Thus the piano is tuned to a frequency of about 441.43 Hz.

Headstart for next week, Week 08, starting Monday 2004/11/08:
– Read Chapter 12 “Continuous Distribution s of Coherent Sources; the Fraunhofer Approximation” in Towne, omit 12-15
– – Section 12-1 “Introduction”
– – Section 12-2 “Radiation pattern from coherent sources continuously distributed along a line segment”
– – Section 12-3 “The Fraunhofer approximation”
– – Section 12-4 “Study of the Fraunhofer pattern”
– – Section 12-5 “Vibration curve for the Fraunhofer approximation”
– – Section 12-6 “Diffraction by an extremely narrow slit”
– – Section 12-7 “Diffraction by an extremely long slit”
– – Section 12-8 “The Fraunhofer approximation applied to a rectangular distribution of coherent point sources”
– – Section 12-9 “Diffraction by a rectangular aperture”