Physics 380H - Wave Theory | **Homework #01 - Solutions**

Fall 2004 | Due 12:01 PM, Monday 2004/09/20

[35 points total]

"Journal" questions:
– What are your goals for the course? What are your expectations for the course?
– Any comments about this week's activities? Course content? Assignment? Lab?

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1. Send me (jbeda@trentu.ca) an e-mail message from your trentu.ca account, with a subject of "380H–HW–01" [5]

   Solution: Send the email message.

2. Sign onto WebCT and post a message in the discussion forum "General social discussions". [5]

   Solution: Post the WebCT message.

3. Put your name and email address and phone number inside your texts and on your calculator and anything else you might misplace - it will not prevent theft, but it will allow anyone who finds your stuff to have a chance of returning it. [0]

   Solution: Break out the pen/pencil and inscribe.

4. For an open flask of volume V and neck length l, with a cross sectional area A, containing air of densityρ, vibrating as sound in the neck, the equation for SHM of the air is:

   $$\rho A l \ddot{x} + \frac{\gamma \rho A^2}{V} x = 0.$$

   What is the angular frequency ω of the sound? [5]
   Solution: The general form of the equation for SHM is:

   $$\ddot{x} = -\omega^2 x,$$

   so rewriting the given equation gives:

   $$\ddot{x} = -\frac{\gamma \rho A^2}{\rho A l V} x = -\frac{\gamma A}{l V} x,$$

   thus

   $$\omega = \sqrt{\frac{\gamma A}{l V}}.$$

   The units of ω are rad sec^{-1}.

5. For a small object of mass m, swinging on a string of length l, show that the period of oscillation is

$$T = 2\pi\sqrt{\frac{l}{g}}$$

by applying Newton's laws to arrive at the SHM equation. What approximation must be made? [10]

Solution: The force in the direction of motion (along the arc of the swing) is the component of the force of gravity in that direction:

$$F = -mg\sin\theta.$$

From Newton's Law, the force is related to the acceleration by:

$$F = ma = m\frac{\mathrm{d}^2 s}{\mathrm{d}t^2}.$$

Additionally, the arc length s is related to the angle θ by

$$s = l\theta, \frac{\mathrm{d}^2 s}{\mathrm{d}t^2} = l\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}.$$

Putting them together gives us:

$$-mg\sin\theta = ml\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2},$$
$$-g\sin\theta = l\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2},$$
$$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = \frac{g}{l}sin\theta.$$

In order for this to be SHM, we must have that $\sin\theta = \theta$. Fortunately, for small angles this is a fairly good approximation, so with this assumption we have:

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} = \frac{g}{l}\theta.$$

Thus we have

$$\omega^2 = \frac{g}{l},$$

which gives us

$$\omega = \sqrt{\frac{g}{l}}.$$

Since the period T is related to the angular frequency by

$$T = \frac{2\pi}{\omega},$$

we have

$$T = \frac{2\pi}{\sqrt{\frac{g}{l}}} = 2\pi\sqrt{\frac{l}{g}},$$

with the approximation that $\sin\theta \approx \theta$.

6. (From Towne P1-1. pg 17) Which of the following are solutions to the one-dimensional wave equation for transverse waves on a string? Why or why not? [10]

$$\text{a) } x^2 - 2cxt + c^2t^2 \quad \text{b) } 10\left(x^2 - c^2t^2\right) \quad \text{c) } \sigma x^2 + Tt^2$$
$$\text{d) } \sqrt[3]{\sin\left[(x-ct)^3\right]} \quad \text{e) } 2x - 3ct \quad \text{f) } 10(\sin x)(\cos ct)$$

Solution: Direct substitution into the one-dimensional wave equation can verify that a particular function is a solution. Alternatively if the function is of the form $f(x-ct)+g(x+ct)$, it is a solution. Thus (a), (c), (d), (e), and (f) are all solutions to the one-dimensional wave equation, while (b) is not.

Headstart for next week, Week 02, starting Monday 2004/09/20:
– Read Chapter 1 "Waves" in "Simple Nature" by Crowell
– Review the use of complex numbers and sinusoidal functions, Towne, Appendix I
– Read Chapter 1 "Transverse Waves on a String" in "Wave Phenomena" by Towne
– – Section 1-1 "Introduction"
– – Section 1-2 "Derivation of the wave equation"
– – Section 1-3 "Solution of the one-dimensional wave equation"
– – Section 1-4 "Wave propagation velocity on a string"
– – Section 1-5 "The most general solution to the one-dimensional wave equation"
– – Section 1-6 "Kinematics associated with the waveform"
– – Section 1-7 "Description of a sinusoidal progressive wave"
– – Section 1-8 "Initial conditions applied to the case of a string of infinite length"
– Read Chapter 2 "The Acoustic Plane Wave" in "Wave Phenomena" by Towne, omit 2-6
– – Section 2-1 "Definition of the variables"
– – Section 2-2 "Derivation of the wave equation"
– – Section 2-3 "The velocity of sound"