TRENT UNIVERSITY

Faculty of Arts and Science

Final Examination

Physics 20

1968

Information needed: Physical constants, table of natural logarithms

Part A. Answer 2 of the following 3 questions.

1. A certain substance obeys the equation of state $pv^2 = RT$.

   Show a process on a p - v diagram in which the volume increases from v_o to $3v_o$ at constant temperature. Calculate the work done on the gas during this process.

   Derive expressions for the compressibility $\varkappa$ and coefficient of thermal volume expansion β, and give an equation for the work done by the gas during an infinitesimal change of state variables in terms of $\varkappa$ and β.

2. (a) State the first law of thermodynamics.

   (b) Using an ideal gas as working substance, show that the efficiency of a Carnot engine is a function only of the reservoir temperatures T_1 and T_2.

3. (a) State the Kelvin and Clausius forms of the second law of thermodynamics, and show that they are equivalent.

(b) Two one-kilogram reservoirs of water at 20°C and 80°C are mixed together in a well-insulated vessel. What is the entropy change of the water, and of the surroundings?

Part B. Answer 4 of the following 5 questions.

1. (a) The energy in light rays arrives in bundles, called "photons". Explain how the photoelectric effect demonstrates this.

(b) A metal whose work function is 3 volts is illuminated by light of wavelength 2000 Å. Calculate the velocity of the most energetic electrons ejected from the metal.

2. (a) Sketch a graph showing the variation of nuclear binding energy per nucleon with atomic weight, and show how the liquid-drop model explains the main features of this curve.

(b) Explain, using the graph of part (a), how nuclear fusion and fission lead to the release of large amounts of energy, and indicate some of the technological problems involved in harnessing these processes.

3. (a) A target emits strong X-rays at wavelengths of 1.0 and 1.2 Å when bombarded with electrons. It is desired to obtain only the rays at 1.2 Å by passing the beam through a thin filter of another material. Discuss the properties required by the atoms of the filter material.

(b) Derive the Bragg law $n\lambda = 2d \sin\theta$, and show how this relation may be used to investigate the spectral distribution of intensity (variation of intensity with wavelength) in an X-ray beam.

4. (a) Show how the Schrödinger equation may be derived from the classical equation of motion of a particle by applying Schrödinger's prescription to find a quantum mechanical operator.

(b) A particle of mass m is confined to a range on the x axis between x = 0 and x = a where its potential V = 0. Find acceptable eigenfunctions and eigenvalues to describe its state.

5. (a) State the two basic postulates of the special theory of relativity.

(b) Derive, from the Lorentz-Einstein equations, the velocity v'_x of a particle measured in a system S' moving at velocity v in the x direction relative to a system S in which the velocity of the particle is v_x.

(see next page)

(c) An electron is bent to a radius of 1 meter in a magnetic field of 0.01 webers/m^2. Find the mass of this electron in terms of its rest mass.

Trent University Final Examination

PHYSICS 202A

Tuesday, December 11, 2000, 3:30–6:30 p.m. OCA 207

Data sheets are provided. Students may use electronic calculators.

In addition to the information on the data sheets, the following formulae may be useful in some problems.

Lorentz Transformation Equations

$$x' = \gamma(v)(x - vt) \qquad y' = y \qquad z' = z \qquad t' = \gamma(v)(t - vx/c^2)$$

Doppler shift. If a source of light emits light with frequency ν_0 measured in the rest frame of the source, then when the source and an observer are approaching one another or receding from one another at speed v, the observer measures the light frequency to be

$$\nu = \nu_0\sqrt{\frac{1+v/c}{1-v/c}}$$

where v is positive if the source and observer are approaching one another and v is negative if the source and observer are receding from one another.

One-electron atoms. The allowed energy values for the electron in a one-electron atom with a nucleus having electric charge $+Ze$ are given by

$$E_n = -\frac{\mu Z^2 e^4}{2\,(4\pi\epsilon_0)^2\hbar^2\,n^2} = -\frac{13.60\ Z^2}{n^2}\ \text{eV}, \qquad n = 1, 2, 3, 4, \ldots$$

where μ is the reduced mass of the electron $\mu = (m_e \times m_{nucleus})/(m_e + m_{nucleus})$.

Schrödinger's Time-independent Equation for a particle of mass m, with potential energy $U(x)$, energy E:

$$\frac{-\hbar^2}{2m}\nabla^2\psi(x) + U(x)\psi(x) = E\psi(x)$$

Note:

$$\sin^2\theta = \frac{1}{2}(1-\cos 2\theta) \qquad \int \sin^2 ax\,dx = \frac{x}{2} - \frac{\sin 2ax}{4a} \qquad \int x\sin^2 ax\,dx = \frac{x^2}{4} - \frac{x\sin 2ax}{4a} - \frac{\cos 2ax}{8a^2}$$

PART A. [40 marks] **Answer BOTH questions.**

1. Consider a particle of mass m, energy E, confined in a one-dimensional "box" with rigid walls. The potential energy function is

$$U(x) = \begin{cases} \infty & \text{for } x \leq 0 \\ 0 & \text{for } 0 \leq x \leq L \\ \infty & \text{for } x \geq L \end{cases}$$

(a) Set up and solve Schrödinger's equation and show that the eigenfunctions must have the form

$$\psi_n(x) = \begin{cases} A_n \sin(n\pi x/L) & \text{for } 0 \leq x \leq L \\ & \text{where } n = 1, 2, 3, \ldots \\ 0 & \text{for } x \leq 0,\ x \geq L \end{cases}$$

(b) Show that $A_n = \sqrt{2/L}$ is required for $\psi_n(x)$ to be normalized.

Note: $\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$

2. A certain particle of mass m is described by the time-independent wave function

$$\psi(x) = \begin{cases} \sqrt{\frac{30}{L^5}}(x^2 - Lx) & 0 \leq x \leq L \\ 0 & x < 0\ , x > L \end{cases}$$

(a) What physical situation corresponds to this wave function?

(b) Show that $\psi(x)$ is normalized to unity.

(c) Calculate the expectation value of x.

(d) Calculate the expectation value of p_x.

Part B. [60 marks]. Answer any THREE questions.

3. (a) A hydrogen atom ($Z = 1$) in its *first excited state* is approaching another hydrogen atom in its *ground state* at a relative speed of $0.1683c$. The excited atom emits a photon, which travels toward the atom which is in its ground state.
If the second atom absorbs the photon, what energy level will it be left in?

   (b) What photon wavelength(s) may be emitted if the atom returns to its ground state?

4. (a) A hydrogen atom in its first excited state returns to its ground state. Determine whether or not the emitted photon is capable of ejecting a photoelectron from an aluminum surface (work function ϕ=4.22 eV).

   (b) Calculate the maximum kinetic energy of the photoelectrons produced when light of wavelength 200 nm falls on an aluminum surface (work function given in part (a)).
(*Note*: 1 nm $= 10^{-9}$ m.)

   (c) Determine whether or not the photoelectrons of part (b) can cause excitation of a normal hydrogen atom (in its ground state) to a higher energy state. What is the highest energy level that can be excited?

   (d) Calculate the de Broglie wavelength of the photoelectrons of part (b). How might this wavelength be measured?

5. In Compton scattering of photons of wavelength λ by free electrons (mass m_e), the wavelength of the scattered photon is given by

$$\lambda' = \lambda + \frac{h}{m_e c}(1 - \cos\phi)$$

   where ϕ is the angle between the direction of the incident photon and the direction of the scattered photon, h is Planck's constant, and c is the speed of light.

   (a) If the incident photon has energy 80 keV, and the electron recoils with kinetic energy of 6 keV, find the angle ϕ.

   (b) Find the angle θ at which the electron recoils.

   (c) Find the *maximum* recoil kinetic energy that the electron could acquire.

6. Gamma-rays are ejected from cesium nuclei with an energy of 0.662 MeV. One of these gamma-rays undergoes Compton scattering through an angle of 120°.

(a) Calculate the kinetic energy, momentum and recoil angle of the recoil electron. (See problem 5 for the Compton scattering formula.)

(b) Calculate the de Broglie wavelength of the recoil electron.

(c) Would the scattered photon be capable of producing an electron-positron pair? If not, why not?

7. (a) The (constant) probability per unit time of radioactive disintegration of an unstable atom is λ. If N_0 of these atoms are present at time $t = 0$, show that the number of the unstable atoms remaining after a time t is

$$N(t) = N_0 e^{-\lambda t}$$

(b) Show that the half-life (time after which half of the atoms have decayed) is

$$T_{1/2} = \frac{\ln 2}{\lambda}$$

(c) An average 70 kg person contains 140 g of potassium, of which 0.0119% is ${}^{40}_{19}\mathrm{K}$, which is radioactive. The half-life of ${}^{40}_{19}\mathrm{K}$ is 1.3×10^9 years.

How many radioactive decays of ${}^{40}_{19}\mathrm{K}$ occur in the average 70 kg person in

(i) one second?

(ii) the average lifespan of 70 years?

Note: The atomic weight of ${}^{40}_{19}\mathrm{K}$ is 39.964.

FUNDAMENTAL CONSTANTS

Quantity	Symbol	Value
Elementary Charge	e	1.6022×10^{-19} C
Speed of light in vacuum	c	$2.9979\times10^{8}\mathrm{m\cdot s^{-1}}$
Avogadro's number	N_A	$6.0221\times10^{23}\mathrm{mol^{-1}}$
Planck's constant	h	$6.6261\times10^{-34}\mathrm{J}\cdot s$
Boltzmann constant	k	$1.3807\times10^{-23}\mathrm{J\cdot K^{-1}}$
Atomic mass unit	u	1.6605×10^{-27} kg 931.49 MeV/c^2
Permittivity of vacuum	ϵ_0	$8.8542\times10^{-12}\mathrm{F\cdot m^{-1}}$
Permeability of vacuum	μ_0	$4\pi\times10^{-7}\mathrm{N\cdot A^{-2}}$
Stefan-Boltzmann constant	σ	5.6708×10^{-8} W·m^{-2}· K^{-4}
Gravitation constant	G	$6.6726\times10^{-11}\mathrm{N\cdot m^2\cdot kg^{-2}}$

SOME PARTICLE MASSES

Particle	Mass in Units of		
	kg	MeV/c^2	u
Electron	9.1094×10^{-31} kg	0.51100	5.4858×10^{-4}
Muon	1.8835×10^{-28}	105.66	0.11343
Proton	1.6726×10^{-27}	938.27	1.00728
Neutron	1.6749×10^{-27}	939.57	1.00866
Deuteron	3.3436×10^{-27}	1875.61	2.01355
α particle	6.6447×10^{-27}	3727.38	4.00151

CONVERSION FACTORS

1 yr=3.156×10^{7} s	1T=10^4G
1 lightyear=9.461×10^{15} m	1 Ci=3.7×10^{10} Bq
1 cal=4.186 J	1 barn=$10^{-28}\mathrm{m}^2$
1 MeV/c=$5.344\times10^{-22}\mathrm{kg\cdot m/s}$	1 u = 1.66054×10^{-27} kg
1 eV=1.6022×10^{-19}J	

USEFUL COMBINATIONS OF CONSTANTS

$\hbar = h/2\pi = 1.0546\times10^{-34}\mathrm{J\cdot s} = 6.5821\times10^{-16}\mathrm{eV\cdot s}$

$\hbar c = 3.1615\times10^{-26}\mathrm{J\cdot m} = 197.33\mathrm{ev\cdot nm}$

Compton wavelength $\lambda_C = \frac{h}{m_e c} = 2.4263\times10^{-12}\mathrm{m}$

Fine structure constant $\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} = 0.0072974 \simeq \frac{1}{137}$

$hc = 1.9864\times10^{-25}\mathrm{J\cdot m} = 1239.8\mathrm{eV\cdot nm}$

$\frac{1}{4\pi\epsilon_0} = 8.9876\times10^{9}\mathrm{N\cdot m^2\cdot C^{-2}}$

$\frac{e^2}{4\pi\epsilon_0} = 2.3071\times10^{-28}\mathrm{J\cdot m} = 1.4400\mathrm{eV\cdot nm}$

$kT = 2.5248\times10^{-2}\mathrm{eV} \simeq \frac{1}{40}eV$ at T -293 K

Hydrogen ground state $E_0 = \frac{e^2}{8\pi\epsilon_0 a_0} = 13.606\mathrm{eV}=2.1799\times10^{-18}$ J

Rydberg constant $R_\infty = \frac{E_0}{hc} = 1.09737\times10^{7}\mathrm{m^{-1}}$

Bohr radius $a_0 = \frac{4\pi\epsilon_0\hbar^2}{m_e e^2} = 5.2918\times10^{-11}\mathrm{m}$

Hydrogen Rydberg $R_H = \frac{\mu}{m_e}R_\infty = 1.09678\times10^{7}\mathrm{m^{-1}}$

Classical electron radius=$\alpha^2 a_0 = 2.8179\times10^{-15}\mathrm{m}$

Bohr magneton $\mu_B = \frac{e\hbar}{2m_e} = 9.2740\times10^{-24}\mathrm{J/T}=5.7884\times10^{-5}\mathrm{eV/T}$

Nuclear magneton $\mu_N = \frac{e\hbar}{2m_p} = 5.0508\times10^{-27}\mathrm{J/T}=3.1525\times10^{-8}$ eV/T

Magnetic flux quantum $\Phi_0 = \frac{h}{2e} = 2.0678\times10^{-15}$ T·m^{-2}

Gas constant $R = N_A k = 8.3145\ \mathrm{J\cdot mol^{-1}\cdot K^{-1}}$