Physics 380H - Wave Theory **Homework #11**

Fall 2004 Due 12:01 PM, Monday 2004/12/06

[35 points total]

"Journal" questions:
– How did the expectation for the course match with how the course actually went? Did you meet your own goals for the course? Did your goals or expectations for the course change through the semester? In what ways?
– Any comments about this week's activities? Course content? Assignment? Lab?

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1. (From Towne P15-5, pg 373) The initial conditions for a string with two fixed ends are

$$y(x,0) = 0, \qquad \text{and} \qquad \dot{y}(x,0) = \sin(2\pi x/l).$$

It is clear that these initial conditions correspond to a particular choice of amplitude and phase of the first harmonic. Show that the formal machinery of the normal-modes expansion leads to the conclusion that this is the only participating mode and exhibit the resulting solution for $y(x,t)$. [10]

2. (From Towne P15-7, pg 373) A string with two fixed ends is plucked at the centre. Assume that the string is of length l and is at rest at $t = 0$ and that the initial profile is triangular of height h:

$$y(x,0) = \begin{cases} \dfrac{2h}{l}x, & x \leq \dfrac{l}{2}, \\ \dfrac{2h}{l}(l-x), & \dfrac{l}{2} > x. \end{cases}.$$

Show that the even harmonics will be missing and that the expansion coefficients are $a_n = 0$ and

$$b_n = (-1)^m \frac{8h}{(2m+1)^2\pi^2}$$

for $n = 2m + 1$. [10]

3. (From Towne P15-10, pg 374) A string of length l is fixed at both ends. If all points on the string are initially at rest and the initial shape of the string is specified by $y(x,0) = x(\sin kx)$, where $k = \pi/l$,
   (a) find the coefficients in the Fourier series representation of this function. [5]
   (b) Draw a bar graph indicating the relative energies of the first few modes. [5]
   (c) Graph the function $y(x,0)$ and compare this with a graph of the sum of the *two* most prominent modes in the Fourier series at $t = 0$. [5]

Headstart for next week, Week 12, starting Monday 2004/12/06:
– Read Chapter 15 "Waves Confined to a Limited Region" in Towne, omit 15-14, 15-15
– – Section 15-12 "Forced motion of a string"
– – Section 15-13 "Eigenfrequencies as resonance frequencies of a string driven sinusoidally at one end"
– – Section 15-16 "Normal modes of a uniformly stretched rectangular membrane"
– – Section 15-17 "Fourier integral analysis on a semi-infinite string"
– – Section 15-18 "Fourier analysis over the whole x-axis"
– Review notes, review texts, review assignments, learn material, do well on exam