Physics 202H - Introductory Quantum Physics I
Midterm Test - A - Solutions

Fall 2004 **Name:** ____________________

Thursday 2004/11/04 **Student Number:** ____________________

This examination paper includes 2 pages and 10 questions. You are responsible for ensuring that your copy of the paper is complete. Bring any discrepancy to the attention of your invigilator.

Special Instructions:

The only aids allowed are: a one (1) page single-sided hand-written formula sheet, and a calculator. When completed, turn in all exam booklets, the test paper, and the formula sheet.

Write your name and student number on the top of this paper AND on the front of your answer booklet AND on your formula sheet. Be prepared to present your student ID for verification.

Portable communications devices of all types (e.g. pagers, cellular phones, communicating calculators) are prohibited in the examination room. All such devices must be turned off prior to the start of the examination. A penalty of 5% of the exam mark may be assessed to anyone who fails to prevent a call from interrupting the examination.

Giving or receiving aid during an exam is a violation of university rules and may result in a failing grade and/or expulsion from the university.

Answer questions one through five on the test paper, and the remaining questions in your exam booklet(s).

1. The "old quantum theory" can only be applied to atoms of hydrogen and helium. [2]

   (a) True (b) **False**

2. An object at 300°K, at equilibrium with room temperature [3]

   (a) **emits radiation to the room and absorbs an equal amount from the room.**

   (b) no longer emits any radiation

   (c) absorbs more energy than it emits if it is blacker than the room.

3. When light of wavelength 500 nm shines on a metal surface the maximum kinetic energy of electrons ejected by the photoelectric effect is 1.48 eV. What is the maximum kinetic energy of electrons ejected from the surface if one uses light with a wavelength of 400 nm? [3]

   (a) 1.48 eV

   (b) **2.10 eV**

   (c) 0.86 eV

4. Rutherford's scattering of alpha particles from a thin foil target had the surprising result that [3]

   (a) many of the alpha particles were almost completely undeflected.

   (b) the alpha particles were actually helium nuclei.

   (c) **some alpha particles were deflected at great angles, up to 180°.**

5. If an electron and a photon in free space each have a wavelength of 600 nm, [3]

   (a) the kinetic energy of the electron is greater than that of the photon.

   (b) the kinetic energy of the electron is equal to that of the photon.

   (c) **the kinetic energy of the electron is less than that of the photon.**

6. He^+ is the symbol for singly ionized helium. The He^+ nucleus has a charge, Z, of 2. One electron remains, one electron having been removed.

   (a) What is the energy of the second excited state ($n = 3$) of He^+? [5]

   Solution: The Bohr atomic model gives us an expression for the energy of the atom, namely:

$$E_n = -\left(\frac{1}{4\pi\epsilon_0}\right)^2 \frac{m_e e^4 Z^2}{2\hbar^2 n^2} = -(13.6\,\text{eV})\frac{Z^2}{n^2} = -(2.17\times10^{-18}\,\text{J})\frac{Z^2}{n^2}$$
$$E_3 = -(13.6\,\text{eV})\frac{2^2}{3^2} = -(13.6\,\text{eV})\frac{4}{9}$$
$$\approx -(6.04\,\text{eV}) \approx -(9.68\times10^{-19}\,\text{J}).$$

   The energy of the second excited state of He^+ is about $-6.04\,\text{eV}$ or $-9.68\times10^{-19}\,\text{J}$.

   (b) What is the energy of the lowest energy transition in the Balmer series of He^+? Recall that the Balmer series has $n_f = 2$. [5]

   Solution: For the Balmer series, $n_f = 2$, so the lowest energy transition will be from $n_i = 3$, as any other transition to the n_f state would be greater in magnitude, thus

$$\Delta E = E_f - E_i = -\left(\frac{1}{4\pi\epsilon_0}\right)^2 \frac{m_e e^4 Z^2}{2\hbar^2}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$
$$= -(13.6\,\text{eV})2^2\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = -(13.6\,\text{eV})\frac{5}{9}$$
$$\Delta E = E_2 - E_3 \approx -(7.56\,\text{eV}) \approx -(1.21\times10^{-18}\,\text{J}),$$

   It would take about $7.56\,\text{eV}$ or $1.21\times10^{-18}\,\text{J}$ to move an electron from the $n = 2$ state to the $n = 3$ state. Equivalently, about $7.56\,\text{eV}$ or $1.21\times10^{-18}\,\text{J}$ is released when an electron moves from the $n = 3$ state to the $n = 2$ state.

7. A 100 watt light bulb produces only about 5 W of visible light. How many visible photons are produced each second, assuming that all of the photons have a wavelength of 550 nm? [5]

   Solution: The power $P = 5\,\text{W} = 5\,\text{J/s}$ is made up of a certain rate R of photons, each having energy given by $E_\gamma = hc/\lambda$. The total number of photons is thus given by dividing the power by the energy per photon.

$$P = RE_\gamma$$
$$R = \frac{P}{E_\gamma} = \frac{P\lambda}{hc}$$
$$= \frac{(5\,\text{J/s})(550\times10^{-9}\,\text{m})}{(6.6260\ldots\times10^{-34}\,\text{J}\cdot\text{s})(2.99792458\times10^{8}\,\text{m/s})}$$
$$= 1.3843\ldots\times10^{19}\,\text{s}^{-1}$$
$$\approx 1.38\times10^{19}\,\text{s}^{-1}$$

   So there are about 1.38×10^{19} photons produced each second.

8. Suppose light of wavelength 662 nm is used to determine the position of an electron to within the wavelength of the light. What will be the minimum resulting uncertainty in the electron's velocity? [5]

Solution: The uncertainty in position is about $\Delta x = 662\,\text{nm}$, so using the Heisenberg relation we can find the uncertainty in momentum, and from there the uncertainty in the velocity.

$$\begin{aligned}
\Delta x \Delta p &> \frac{\hbar}{2} = \frac{h}{4\pi} \\
\Delta p &> \frac{\hbar}{2\Delta x} = \frac{h}{4\pi \Delta x} \\
m\Delta v &> \frac{\hbar}{2\Delta x} = \frac{h}{4\pi \Delta x} \\
\Delta v &> \frac{\hbar}{2m\Delta x} = \frac{h}{4\pi m \Delta x} \\
&> \frac{(6.6260\ldots \times 10^{-34}\,\text{J}\cdot\text{s})}{4\pi(9.109\ldots \times 10^{-31}\,\text{kg})(662 \times 10^{-9}\,\text{m})} \\
&> (87.437\ldots\ \text{m/s}) \approx (87.4\,\text{m/s}).
\end{aligned}$$

The uncertainty in the electrons velocity is greater than about $87.4\,\text{m/s}$.

9. A gamma ray with wavelength of 0.00188 nm is scattered from a free electron by 90°.

(a) What is the Compton wavelength shift after scattering? [5]

Solution: $\cos\theta = 0$ for $\theta = 90°$ so

$$\begin{aligned}\Delta\lambda &= \lambda_c(1-\cos\theta) = \frac{h}{m_0c}(1-\cos\theta)\\ &= \frac{(6.6260\ldots\times10^{-34}\,\mathrm{J\cdot s})}{(9.109\ldots\times10^{-31}\,\mathrm{kg})(2.99792458\times10^{8}\,\mathrm{m/s})}(1-0)\\ &= (2.4263\ldots\times10^{-12}\,\mathrm{m})\\ &\approx (2.43\times10^{-12}\,\mathrm{m}).\end{aligned}$$

The gamma ray's wavelength will *increase* by about 2.43×10^{-15} m after the scattering.

(b) What is the gamma ray's initial energy? [5]

Solution: $E_{\gamma i} = h\nu_{\gamma i} = hc/\lambda_{\gamma i}$ so

$$\begin{aligned}E_{\gamma i} &= h\nu_{\gamma i} = \frac{hc}{\lambda_{\gamma i}}\\ &= \frac{(6.6260\ldots\times10^{-34}\,\mathrm{J\cdot s})(2.99792458\times10^{8}\,\mathrm{m/s})}{(1.88\times10^{-12}\,\mathrm{m})}\\ &= (1.05662\ldots\times10^{-13}\,\mathrm{J})\\ &\approx (1.06\times10^{-13}\,\mathrm{J}) \approx (0.659\,\mathrm{MeV})\end{aligned}$$

(c) What kinetic energy is given to the recoiling electron? [5]

Solution: The electron's kinetic energy gain will be equal to the loss of energy by the photon due to its increased wavelength. $E_{\gamma f} = h\nu_{\gamma f} = hc/\lambda_{\gamma f} = hc/(\lambda_{\gamma i}+\Delta\lambda)$ so

$$\begin{aligned}E_{\gamma f} &= h\nu_{\gamma f} = \frac{hc}{\lambda_{\gamma f}} = \frac{hc}{\lambda_{\gamma i}+\Delta\lambda}\\ &= \frac{(6.6260\ldots\times10^{-34}\,\mathrm{J\cdot s})(2.99792458\times10^{8}\,\mathrm{m/s})}{(1.88\times10^{-12}\,\mathrm{m})+(2.4263\ldots\times10^{-12}\,\mathrm{m})}\\ &= (4.6128\ldots\times10^{-14}\,\mathrm{J}) = (0.2879\ldots\,\mathrm{MeV})\\ K_e &= -\Delta E = -E_{\gamma f}+E_{\gamma i} = hc\left(\frac{1}{\lambda_{\gamma i}} - \frac{1}{\lambda_{\gamma i}+\Delta\lambda}\right) = \frac{hc\Delta\lambda}{\lambda_{\gamma i}(\lambda_{\gamma i}+\Delta\lambda)}\\ &= -(4.6128\ldots\times10^{-14}\,\mathrm{J}) + (1.05662\ldots\times10^{-13}\,\mathrm{J})\\ &= (5.953\ldots\times10^{-14}\,\mathrm{J})\\ &\approx (5.95\times10^{-14}\,\mathrm{J}) \approx (0.372\,\mathrm{MeV})\end{aligned}$$

The electron's kinetic energy after the collision is about $5.95\times10^{-14}\,\mathrm{J} \approx 0.372\,\mathrm{MeV}$. Note that while this energy is close to the rest mass energy of the electron, since the rest mass energy of the electron does not change, it does not enter into the calculation, and while this system is relativistic, since we only are interested in the changes in kinetic energy, we do not have to make explicit use of any special relativistic forms of energy.

10. Given a material's cross section σ and the number of atoms per unit volume ρ, what thickness t of material would be needed so that incoming photons would have about a 10% chance of interacting? [5]

Solution: The thickness that gives this chance of interacting can be found using the intensity relationship, noting that if 10% of the incident particles interact, then 90% of them remain to continue on, so $I_f = 0.9 I_i$:

$$\begin{aligned}
I_f = I_i \mathrm{e}^{-\sigma\rho t} \quad &\Longrightarrow \quad \frac{I_f}{I_i} = \mathrm{e}^{-\sigma\rho t} \\
\frac{I_i}{I_f} = \mathrm{e}^{\sigma\rho t} \quad &\Longrightarrow \quad \ln\left(\frac{I_i}{I_f}\right) = \sigma\rho t \\
t = \frac{1}{\sigma\rho}\ln\left(\frac{I_i}{I_f}\right) &= \frac{1}{\sigma\rho}\ln\left(\frac{10}{9}\right) \\
&= \frac{(0.10536\ldots)}{\sigma\rho} \approx \frac{(0.11)}{\sigma\rho}
\end{aligned}$$

$$\begin{aligned}
c &= 2.99792458 \times 10^{8}\,\mathrm{m/s} \\
e &= 1.602176462 \times 10^{-19}\,\mathrm{Coul} \\
h &= 6.626068 \times 10^{-34}\,\mathrm{J\cdot s} = 4.1356668 \times 10^{-15}\,\mathrm{eV\cdot s} \\
\hbar &= 1.05457148 \times 10^{-34}\,\mathrm{J\cdot s} = 6.58211814 \times 10^{-16}\,\mathrm{eV\cdot s} \\
m_e &= 9.10938188 \times 10^{-31}\,\mathrm{kg} = 0.510998903\,\mathrm{MeV}/c^2 \\
m_p &= 1.67262158 \times 10^{-27}\,\mathrm{kg} = 938.271996\,\mathrm{MeV}/c^2 \\
m_n &= 1.6749286 \times 10^{-27}\,\mathrm{kg} = 939.565630\,\mathrm{MeV}/c^2 \\
\frac{1}{4\pi\epsilon_0} &= 8.988 \times 10^{9}\,\mathrm{N\cdot m^2/Coul^2}
\end{aligned}$$