TRENT UNIVERSITY
DEPARTMENT OF PHYSICS
PHYSICS 202H FINAL EXAMINATION

December 10, 2002 Time: 3 hours

PART A

Answer two (2) questions.

1. Use the fact that $\rho_T(\lambda) = 4R_T(\lambda)/c$ to estimate the pressure due to radiation at all wavelengths in the core of a massive star, where $T = 5 \times 10^7$ K. Express the result in units of atmospheric pressure at Earth's surface, *i.e.*, 1.01325×10^5 N m^{-2}.

2. A photon of energy $m_e c^2$ (where m_e is the electron rest mass) undergoes Compton scattering from an electron. Find the energy of a photon scattered through 60° , the recoil momentum of the electron (keV/c), and the recoil angle of the electron.

3. Consider an electron located somewhere within an atom of diameter 1 Å. What is the uncertainty in its momentum? Is this consistent with the fact that the binding energies of atoms are of the order of 10 eV? On the basis of this sort of argument, would you expect to find electrons within nuclei, which have diameters $\sim 2 \times 10^{-15}$ m? (The binding energy of a typical nucleon is of the order of a few MeV.)

4. Calculate (in the framework of the Bohr model) the speed of the *proton* in the ground state and in the first excited state of hydrogen.

PART B

Answer question 5 and one other question.

5. Quantization in circular orbits is achieved by combining the equation of motion

$$\frac{mv^2}{r} = \left| \frac{dV(r)}{dr} \right|,$$

(where V is the potential energy) with the angular momentum quantization condition $mvr = n\hbar$. Use this procedure to calculate the spectrum for circular motion in the potential $V(r) = F_0 r$, where F_0 is a constant.

6. By what amount does the wavelength of the $n = 3 \rightarrow 2$ transition in tritium (an isotope of hydrogen with mass number $A = 3$) differ from that of the corresponding transition in ordinary hydrogen?

7. Find the phase velocities of 1 MeV protons and 1 MeV electrons.

8. Electron neutrinos stream from the Sun at the rate of $\sim 3 \times 10^{37}$ s^{-1}. At what rate do they enter a detector of area 50 m^2 on Earth, distant about 1.5×10^{11} from the Sun? Assume that the detector is 5 m thick, and filled with a liquid of density 1000 kg m^{-3}: each molecule of the liquid has a mass of 166 atomic mass units. If 10 neutrinos are captured in the tank every 30 days, what is the capture cross-section? (Assume normal incidence on the top of the tank.)

PART C

Answer one of the following questions.

9. An electron moves in a potential described by

$$V(x) = \begin{cases} 0, & -a/2 \le x \le a/2; \\ \infty, & \text{otherwise} \end{cases}$$

(a) Without referring to the time-independent Schrodinger Equation, find an expression for the energy in the nth state.

(b) The ground-state wavefunction is

$$\Psi(x,t) = \begin{cases} A\cos(\pi x/a)\, e^{-iEt/\hbar}, & -a/2 < x < a/2 \\ 0, & \text{elsewhere,} \end{cases}$$

Normalize $\Psi(x,t)$.

(c) Evaluate $\langle x \rangle$, $\langle x^2 \rangle$, $\langle p \rangle$, $\langle p^2 \rangle$ in the ground state. Then, evaluate the product $\Delta x\, \Delta p$, and verify that the result is consistent with the Heisenberg Uncertainty Principle.

(d) For large values of n, the simple prescription of part (a) fails. Why? Estimate the value of n at which you might expect the problem to become apparent.

10. A particle with energy E is incident on a potential step of height V_0, with $E > V_0$. The step is located at $x = 0$. Find the reflection coefficient. Suppose that the particle is a 20 eV electron, and that $V_0 = 15$ eV. What is the probability that the electron passes over the step?

You may find the following integral useful:

$$\int x^2 \cos^2 bx\, dx = \frac{4\,b^3x^3 + 3\,(2\,b^2x^2 - 1)\sin 2bx + 6\,bx\cos 2bx}{24\,b^3}$$

RECOMMENDED VALUES OF FUNDAMENTAL CONSTANTS

Quantity	Symbol	Value	Units
Speed of light in vacuum	c	299 792 458	$\mathrm{m\ s^{-1}}$
Newtonian constant of gravitation	G	6.673(10)	$10^{-11}\ \mathrm{m^3\ kg^{-1}\ s^{-2}}$
Permeability of vacuum	μ_0	$4\pi \times 10^{-7}$	$\mathrm{N\ A^{-2}}$
Permittivity of vacuum	ϵ_0	$1/\mu_0 c^2$	
		$= 8.854\,187\,817\ldots$	$10^{-12}\ \mathrm{F\ m^{-1}}$
Planck constant	h	6.626 068 76(52)	$10^{-34}\ \mathrm{J\ s}$
		= 4.135 667 27(16)	$10^{-15}\ \mathrm{eV\ s}$
Planck constant $h/2\pi$	$\hbar$	1.054 571 596(82)	$10^{-34}\ \mathrm{J\ s}$
		= 6.582 118 89(26)	$10^{-16}\ \mathrm{eV\ s}$
Boltzmann constant	k	1.380 6503(24)	$10^{-23}\ \mathrm{J\ K^{-1}}$
		= 8.617 342(15)	$10^{-5}\ \mathrm{eV\ K^{-1}}$
Stefan-Boltzmann constant	σ	5.670 400(40)	$10^{-8}\ \mathrm{W\ m^{-2}\ K^{-4}}$
Atomic mass unit	u	1.660 538 73(13)	$10^{-27}\ \mathrm{kg}$
		= 931.494 013(37)	MeV/c^2
Elementary charge	e	1.602 176 462(63)	$10^{-19}\ \mathrm{C}$
Fine structure constant	α	1/137.035 999 76(50)	
Electron mass	m_e	9.109 381 88(72)	$10^{-31}\ \mathrm{kg}$
		= 0.510 998 902(21)	MeV/c^2
Electron classical radius	r_e	2.817 940 285(31)	$10^{-15}\ \mathrm{m}$
Electron Compton wavelength	λ_C	2.426 310 215(18)	$10^{-12}\ \mathrm{m}$
Electron g-factor	g	−2.002 319 304 3737(82)	
Proton mass	m_p	1.672 621 58(13)	$10^{-27}\ \mathrm{kg}$
		= 938.271 998(38)	MeV/c^2
Proton-electron mass ratio	m_p/m_e	1836.152 6675(39)	
Neutron mass	m_n	1.674 927 16(13)	$10^{-27}\ \mathrm{kg}$
		= 939.565 330(38)	MeV/c^2
Neutron-electron mass ratio	m_n/m_e	1838.683 6550(40)	
Bohr radius	a_0	0.529 177 2083(19)	$10^{-10}\ \mathrm{m}$
Rydberg constant	R_∞	1.097 373 156 8549(83)	$10^{7}\ \mathrm{m^{-1}}$
Bohr magneton	μ_B	9.274 008 99(37)	$10^{-24}\ \mathrm{J\ T^{-1}}$
Nuclear magneton	μ_N	5.050 783 17(20)	$10^{-27}\ \mathrm{J\ T^{-1}}$
Avogadro's constant	N_A	6.022 141 99(47)	$10^{23}\ \mathrm{mol^{-1}}$

Useful Combinations of Constants

$$\frac{1}{4\pi\epsilon_0} = 8.987\,551\,796 \times 10^9\ \mathrm{N\ m^2\ C^{-2}}$$

$$\frac{e^2}{4\pi\epsilon_0} = 1.439\,965\,173\ \mathrm{eV\ nm}$$

$$hc = 1239.841\,86\ \mathrm{eV\ nm}$$