Physics 202H - Introductory Quantum Physics I
Homework #11 - Solutions

Fall 2004 Due 5:01 PM, Monday 2004/12/06

[65 points total]

"Journal" questions. Briefly share your thoughts on the following questions:
– How did the expectation for the course match with how the course actually went? Did you meet your own goals for the course? Did your goals or expectations for the course change through the semester? In what ways?
– Any comments about this week's activities? Course content? Assignment? Lab?

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1. (From Eisberg & Resnick, Q 6-22, pg 227) In the $n = 3$ state, the probability density function for a particle in a box is zero at two positions between the walls of the box. How then can the particle ever move across these positions? Limit your discussion to about 50 words or so. [10]

   Solution: This problem is based on a confusion of terms. The probability density function does not really describe a particle that is bouncing between the walls of the box, occupying various positions as it travels back and forth, but rather the probability density function gives a method of finding the probability of measuring the particle in various regions. Thus it is perfectly valid to have regions with non-zero probability of measuring the particle separated by regions with zero probability of measuring the particle. The wave function for a particle in a box does not describe a travelling wave, but rather a standing wave, like a wave on a string, which can (and does) have zero nodes at various positions.

2. (From Eisberg & Resnick, P 6-7, pg 228) Consider a particle passing over a rectangular potential barrier. Write the general solutions, presented in Eisberg & Resnick, Section 6-5, which give the form of ψ in the different regions of the potential.

   (a) Find four relations between the five arbitrary constants by matching ψ and $\mathrm{d}\psi/\mathrm{d}x$ at the boundaries between these regions. (Hint: Use the same notation as Eisberg & Resnick, Section 6-5 for A, B, C, etc. to make it easier to compare results.) [10]

   Solution: The potential energy function $V(x)$ with $E > V_0$ looks like figure 1.

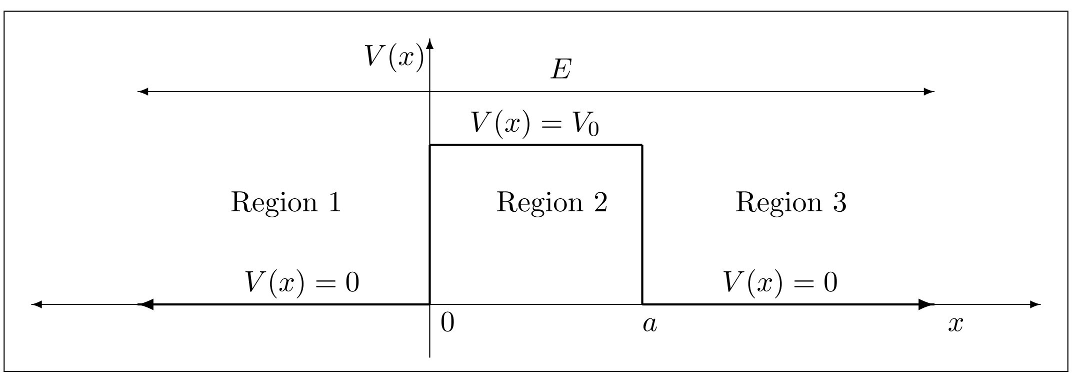

Figure 1: Barrier potential, $E > V_0$

Following Eisberg & Resnick, the time-independant Schroedinger equation has different forms for each region, specifically

$$
\begin{aligned}
-\frac{\hbar^2}{2m}\frac{\mathrm{d}^2\psi_1(x)}{\mathrm{d}x^2} &= E\psi_1(x), && x<0, && \text{Region 1,}\\
-\frac{\hbar^2}{2m}\frac{\mathrm{d}^2\psi_2(x)}{\mathrm{d}x^2} &= (E-V_0)\psi_2(x), && 0<x<a, && \text{Region 2,}\\
-\frac{\hbar^2}{2m}\frac{\mathrm{d}^2\psi_1(x)}{\mathrm{d}x^2} &= E\psi_1(x), && a<x, && \text{Region 3.}
\end{aligned}
$$

Solutions to the TISE in each region are, most generally

$$
\begin{aligned}
\psi_1(x) &= A\mathrm{e}^{\mathrm{i}k_1x}+B\mathrm{e}^{-\mathrm{i}k_1x}, && x<0, && \text{Region 1,}\\
\psi_2(x) &= F\mathrm{e}^{\mathrm{i}k_2x}+G\mathrm{e}^{-\mathrm{i}k_2x}, && 0<x<a, && \text{Region 2,}\\
\psi_3(x) &= C\mathrm{e}^{\mathrm{i}k_3x}+D\mathrm{e}^{-\mathrm{i}k_3x}, && a<x, && \text{Region 3.}
\end{aligned}
$$

where

$$
k_1 = k_3 = \frac{\sqrt{2mE}}{\hbar} \qquad \text{and} \qquad k_2 = \frac{\sqrt{2m(E-V_0)}}{\hbar}.
$$

Note that Eisberg & Resnick use k_{III} which is equal to k_2 above. The derivatives of the functions are

$$
\begin{aligned}
\frac{\mathrm{d}\psi_1(x)}{\mathrm{d}x} &= \mathrm{i}k_1A\mathrm{e}^{\mathrm{i}k_1x}-\mathrm{i}k_1B\mathrm{e}^{-\mathrm{i}k_1x}, && x<0, && \text{Region 1,}\\
\frac{\mathrm{d}\psi_2(x)}{\mathrm{d}x} &= \mathrm{i}k_2F\mathrm{e}^{\mathrm{i}k_2x}-\mathrm{i}k_2G\mathrm{e}^{-\mathrm{i}k_2x}, && 0<x<a, && \text{Region 2,}\\
\frac{\mathrm{d}\psi_3(x)}{\mathrm{d}x} &= \mathrm{i}k_3C\mathrm{e}^{\mathrm{i}k_3x}-\mathrm{i}k_3D\mathrm{e}^{-\mathrm{i}k_3x}, && a<x, && \text{Region 3.}
\end{aligned}
$$

Since ψ and its derivative are finite, single-valued, and continuous, we can apply the following continuity relationships

$$
\left[\psi_1(x)\right]_{x=0} = \left[\psi_2(x)\right]_{x=0}, \qquad \left[\frac{\mathrm{d}\psi_1(x)}{\mathrm{d}x}\right]_{x=0} = \left[\frac{\mathrm{d}\psi_2(x)}{\mathrm{d}x}\right]_{x=0}, \tag{2.01}
$$

$$
\left[\psi_2(x)\right]_{x=a} = \left[\psi_3(x)\right]_{x=a}, \qquad \left[\frac{\mathrm{d}\psi_2(x)}{\mathrm{d}x}\right]_{x=a} = \left[\frac{\mathrm{d}\psi_3(x)}{\mathrm{d}x}\right]_{x=a}. \tag{2.02}
$$

We also know that since there should be no particles coming in from the far right moving in the negative x direction, D must be zero. Applying the continuity relationships (2.01) and (2.02), and dividing through by i where appropriate gives us

$$
A+B = F+G \tag{2.03}
$$

$$
k_1A-k_1B = k_2F-k_2G \tag{2.04}
$$

$$
F\mathrm{e}^{\mathrm{i}k_2a}+G\mathrm{e}^{-\mathrm{i}k_2a} = C\mathrm{e}^{\mathrm{i}k_3a} \tag{2.05}
$$

$$
k_2F\mathrm{e}^{\mathrm{i}k_2a}-k_2G\mathrm{e}^{-\mathrm{i}k_2a} = k_3C\mathrm{e}^{\mathrm{i}k_3a}. \tag{2.06}
$$

Thus equations (2.03), (2.04), (2.05), and (2.06) are the four necessary relationships between the five arbitrary constants A, B, C, F, and G.

(b) Use these relations to evaluate the transmission coefficient T, thereby verifying (Eisberg & Resnick, Equation 6-51, pg 201). (Hint: First eliminate F and G, the amplitudes in the centre region, leaving relations between A, B, and C. Then eliminate B the amplitude of the reflected wave.) [10]

Solution: Multiplying (2.05) by k_2 and adding (2.06) we get

$$\begin{aligned} k_2 F\mathrm{e}^{\mathrm{i}k_2 a} + k_2 G\mathrm{e}^{-\mathrm{i}k_2 a} &= k_2 C\mathrm{e}^{\mathrm{i}k_3 a} \\ k_2 F\mathrm{e}^{\mathrm{i}k_2 a} - k_2 G\mathrm{e}^{-\mathrm{i}k_2 a} &= k_3 C\mathrm{e}^{\mathrm{i}k_3 a} \\ \Rightarrow 2k_2 F\mathrm{e}^{\mathrm{i}k_2 a} + 0 &= (k_2+k_3) C\mathrm{e}^{\mathrm{i}k_3 a} \\ F &= \frac{(k_2+k_3)}{2k_2} C\mathrm{e}^{\mathrm{i}k_3 a}\mathrm{e}^{-\mathrm{i}k_2 a} = \frac{(k_2+k_3)}{2k_2} C\mathrm{e}^{\mathrm{i}(k_3-k_2)a}. \end{aligned} \tag{2.07}$$

Multiplying (2.05) by $-k_2$ and adding (2.06) we get

$$\begin{aligned} -k_2 F\mathrm{e}^{\mathrm{i}k_2 a} - k_2 G\mathrm{e}^{-\mathrm{i}k_2 a} &= -k_2 C\mathrm{e}^{\mathrm{i}k_3 a} \\ k_2 F\mathrm{e}^{\mathrm{i}k_2 a} - k_2 G\mathrm{e}^{-\mathrm{i}k_2 a} &= k_3 C\mathrm{e}^{\mathrm{i}k_3 a} \\ \Rightarrow 0 - 2k_2 G\mathrm{e}^{-\mathrm{i}k_2 a} &= (-k_2+k_3) C\mathrm{e}^{\mathrm{i}k_3 a} \\ G &= \frac{(k_2-k_3)}{2k_2} C\mathrm{e}^{\mathrm{i}k_3 a}\mathrm{e}^{\mathrm{i}k_2 a} = \frac{(k_2-k_3)}{2k_2} C\mathrm{e}^{\mathrm{i}(k_3+k_2)a}. \end{aligned} \tag{2.08}$$

Substituting (2.07) and (2.08) into (2.03) gives us:

$$\begin{aligned} A + B &= \frac{(k_2+k_3)}{2k_2} C\mathrm{e}^{\mathrm{i}(k_3-k_2)a} + \frac{(k_2-k_3)}{2k_2} C\mathrm{e}^{\mathrm{i}(k_3+k_2)a} \\ &= \frac{C\mathrm{e}^{\mathrm{i}k_3 a}}{2k_2}\left[(k_2+k_3)\mathrm{e}^{-\mathrm{i}k_2 a} + (k_2-k_3)\mathrm{e}^{\mathrm{i}k_2 a}\right]. \end{aligned}$$

Since $\mathrm{e}^{-\mathrm{i}k_2 a} = \cos(k_2 a) - \mathrm{i}\sin(k_2 a)$ and $\mathrm{e}^{\mathrm{i}k_2 a} = \cos(k_2 a) + \mathrm{i}\sin(k_2 a)$, we have

$$\begin{aligned} A + B &= \frac{C\mathrm{e}^{\mathrm{i}k_3 a}}{2k_2}\left[(k_2+k_3)(\cos(k_2 a) - \mathrm{i}\sin(k_2 a)) + (k_2-k_3)(\cos(k_2 a) + \mathrm{i}\sin(k_2 a))\right] \\ &= \frac{C\mathrm{e}^{\mathrm{i}k_3 a}}{2k_2}\left[2k_2\cos(k_2 a) - \mathrm{i}2k_3\sin(k_2 a)\right] \\ A + B &= \frac{C\mathrm{e}^{\mathrm{i}k_3 a}}{k_2}\left[k_2\cos(k_2 a) - \mathrm{i}k_3\sin(k_2 a)\right]. \end{aligned} \tag{2.09}$$

Similarly, substituting (2.07) and (2.08) into (2.04) gives us:

$$\begin{aligned} k_1(A - B) &= \frac{k_2(k_2+k_3)}{2k_2} C\mathrm{e}^{\mathrm{i}(k_3-k_2)a} - \frac{k_2(k_2-k_3)}{2k_2} C\mathrm{e}^{\mathrm{i}(k_3+k_2)a} \\ A - B &= \frac{k_2 C\mathrm{e}^{\mathrm{i}k_3 a}}{2k_1 k_2}\left[(k_2+k_3)\mathrm{e}^{-\mathrm{i}k_2 a} - (k_2-k_3)\mathrm{e}^{\mathrm{i}k_2 a}\right] \\ &= \frac{C\mathrm{e}^{\mathrm{i}k_3 a}}{2k_1}\left[(k_2+k_3)\mathrm{e}^{-\mathrm{i}k_2 a} - (k_2-k_3)\mathrm{e}^{\mathrm{i}k_2 a}\right] \\ &= \frac{C\mathrm{e}^{\mathrm{i}k_3 a}}{2k_1}\left[(k_2+k_3)(\cos(k_2 a) - \mathrm{i}\sin(k_2 a)) - (k_2-k_3)(\cos(k_2 a) + \mathrm{i}\sin(k_2 a))\right] \\ &= \frac{C\mathrm{e}^{\mathrm{i}k_3 a}}{2k_1}\left[2k_3\cos(k_2 a) - \mathrm{i}2k_2\sin(k_2 a)\right] \\ A - B &= \frac{C\mathrm{e}^{\mathrm{i}k_3 a}}{k_1}\left[k_3\cos(k_2 a) - \mathrm{i}k_2\sin(k_2 a)\right]. \end{aligned} \tag{2.10}$$

To get rid of B, we can add (2.09) and (2.10) to get

$$\begin{aligned}
A+B &= \frac{C\mathrm{e}^{\mathrm{i}k_3 a}}{k_2}\left[k_2\cos\left(k_2 a\right)-\mathrm{i}k_3\sin\left(k_2 a\right)\right] \\
A-B &= \frac{C\mathrm{e}^{\mathrm{i}k_3 a}}{k_1}\left[k_3\cos\left(k_2 a\right)-\mathrm{i}k_2\sin\left(k_2 a\right)\right] \\
\Rightarrow 2A+0 &= C\mathrm{e}^{\mathrm{i}k_3 a}\left[\left(\frac{k_2}{k_2}+\frac{k_3}{k_1}\right)\cos\left(k_2 a\right)-\mathrm{i}\left(\frac{k_3}{k_2}+\frac{k_2}{k_1}\right)\sin\left(k_2 a\right)\right] \\
A &= \frac{C\mathrm{e}^{\mathrm{i}k_3 a}}{2}\left[\left(\frac{k_2}{k_2}+\frac{k_3}{k_1}\right)\cos\left(k_2 a\right)-\mathrm{i}\left(\frac{k_3}{k_2}+\frac{k_2}{k_1}\right)\sin\left(k_2 a\right)\right] \\
&= \frac{C\mathrm{e}^{\mathrm{i}k_3 a}}{2}\left[\left(\frac{k_1k_2+k_2k_3}{k_1k_2}\right)\cos\left(k_2 a\right)-\mathrm{i}\left(\frac{k_1k_3+k_2k_2}{k_1k_2}\right)\sin\left(k_2 a\right)\right] \\
A &= \frac{C\mathrm{e}^{\mathrm{i}k_3 a}}{2k_1k_2}\left[\left(k_1k_2+k_2k_3\right)\cos\left(k_2 a\right)-\mathrm{i}\left(k_1k_3+k_2k_2\right)\sin\left(k_2 a\right)\right] \qquad (2.11) \\
A^* &= \frac{C^*\mathrm{e}^{-\mathrm{i}k_3 a}}{2k_1k_2}\left[\left(k_1k_2+k_2k_3\right)\cos\left(k_2 a\right)+\mathrm{i}\left(k_1k_3+k_2k_2\right)\sin\left(k_2 a\right)\right].
\end{aligned}$$

At this point we could simplify a bit before finding $T=(k_3C^*C)/(k_1A^*A)$ since we know that $k_3=k_1$, however, if we keep both k_1 and k_3 explicitly in the expression, we will end up with a more general solution that does not depend on having the potential energy in Region 1 equal to the potential energy in Region 3. So using (2.11) we can calculate T^{-1} by

$$\begin{aligned}
T^{-1} = \frac{k_1A^*A}{k_3C^*C} &= \frac{k_1}{4k_1^2k_2^2k_3}\left[\left(k_1k_2+k_2k_3\right)^2\cos^2\left(k_2 a\right)+\left(k_1k_3-k_2k_2\right)^2\sin^2\left(k_2 a\right)\right] \\
&= \frac{1}{4k_1k_2^2k_3}\left[\left(k_1k_2+k_2k_3\right)^2\cos^2\left(k_2 a\right)+\left(k_1k_3+k_2k_2\right)^2\sin^2\left(k_2 a\right)\right]. \qquad (2.12)
\end{aligned}$$

We know that $k_3=k_1$, so putting that into (2.12) gives us

$$\begin{aligned}
T^{-1} &= \frac{1}{4k_1k_2^2k_1}\left[\left(k_1k_2+k_2k_1\right)^2\cos^2\left(k_2 a\right)+\left(k_1k_1+k_2k_2\right)^2\sin^2\left(k_2 a\right)\right] \\
&= \frac{1}{4k_1^2k_2^2}\left[\left(2k_2k_1\right)^2\cos^2\left(k_2 a\right)+\left(k_1^2+k_2^2\right)^2\sin^2\left(k_2 a\right)\right] \\
&= \left[\cos^2\left(k_2 a\right)+\frac{1}{4}\left(\frac{k_1^2+k_2^2}{k_1k_2}\right)^2\sin^2\left(k_2 a\right)\right]. \qquad (2.13)
\end{aligned}$$

With some trig knowledge, we have that $\cos^2\left(k_2 a\right)=1-\sin^2\left(k_2 a\right)$, so (2.13) gives us

$$\begin{aligned}
T^{-1} &= \left[1-\sin^2\left(k_2 a\right)+\frac{\left(k_1^2+k_2^2\right)^2}{4k_1^2k_2^2}\sin^2\left(k_2 a\right)\right] \\
&= \left[1-\frac{4k_1^2k_2^2}{4k_1^2k_2^2}\sin^2\left(k_2 a\right)+\frac{k_1^4+2k_1^2k_2^2+k_2^4}{4k_1^2k_2^2}\sin^2\left(k_2 a\right)\right] \\
&= \left[1+\frac{k_1^4-2k_1^2k_2^2+k_2^4}{4k_1^2k_2^2}\sin^2\left(k_2 a\right)\right] \\
&= \left[1+\frac{1}{4}\left(\frac{k_1^2-k_2^2}{k_1k_2}\right)^2\sin^2\left(k_2 a\right)\right]. \qquad (2.14)
\end{aligned}$$

The values of k_1 and k_2 give us

$$\begin{aligned}
\left(k_1^2 - k_2^2\right)^2 &= \left(\frac{2mE}{\hbar^2} - \frac{2m(E-V_0)}{\hbar^2}\right)^2 = \left(\frac{2mV_0}{\hbar^2}\right)^2 \\
(k_1 k_2)^2 &= \left(\frac{\sqrt{2mE}}{\hbar}\frac{\sqrt{2m(E-V_0)}}{\hbar}\right)^2 = \left(\frac{2m}{\hbar^2}\right)^2 E(E-V_0).
\end{aligned}$$

With these relations (2.14) gives us

$$\begin{aligned}
T^{-1} &= \left[1 + \frac{1}{4}\frac{\left(\frac{2mV_0}{\hbar^2}\right)^2}{\left(\frac{2m}{\hbar^2}\right)^2 E(E-V_0)}\sin^2(k_2 a)\right] \\
&= \left[1 + \frac{1}{4}\frac{V_0^2\sin^2(k_2 a)}{E(E-V_0)}\right] \\
&= \left[1 + \frac{\sin^2(k_2 a)}{4\frac{E}{V_0}\left(\frac{E}{V_0}-1\right)}\right] \\
T &= \left[1 + \frac{\sin^2(k_2 a)}{4\frac{E}{V_0}\left(\frac{E}{V_0}-1\right)}\right]^{-1}. \qquad (2.15)
\end{aligned}$$

Equation (2.15) is the desired Eisberg & Resnick, Equation 6-51.

3. (From Eisberg & Resnick, P 6-9, pg 228) A proton and a deuteron (a particle with the same charge as a proton, but twice the mass) attempt to penetrate a rectangular potential barrier of height 10 MeV and thickness 10^{-14} m. Both particles have total energies of 3 MeV.

   (a) Use qualitative arguments to predict which particle has the highest probability of succeeding. [5]

   Solution: Since they have equal energies, the momentum of the more massive particle will be smaller ($K = p^2/2m$), and the velocity of the less massive particle will be greater ($K = mv^2/2$). With a greater velocity, we might expect that the proton is more likely to penetrate deeper into the barrier, and thus is more likely to make it all the way through. This is also born out by Eisberg & Resnick equations 6-49 and 6-50, where a more massive particle would have a larger value for k_{II} and thus a smaller value for T since $-k_{\text{II}}$ appears in the exponent.

   (b) Evaluate quantitatively the probability of success for both particles.. [10]

   Solution: The values for $k_p a$ and $k_d a$ are

$$\begin{aligned}
k_p a &= \frac{\sqrt{2m_p(V_0-E)}}{\hbar}a & k_d a &= \frac{\sqrt{2m_d(V_0-E)}}{\hbar}a \\
&= \frac{\sqrt{2(938.3\,\text{MeV}/c^2)(7\,\text{MeV})}}{(0.6582\times10^{-15}\,\text{eV}\cdot\text{s})}(10^{-14}\,\text{m}) & &= \frac{\sqrt{2(2m_p)(V_0-E)}}{\hbar}a = \sqrt{2}k_p a \\
k_p a &= 5.80839\ldots & k_d a &= 8.21431\ldots.
\end{aligned}$$

   We need to calculate the transmission coefficient given by Eisberg & Resnick, Equation 6-49 (we cannot really use 6 50 since that is only valid for $k_{\text{II}}a \gg 1$, which it is not in this case.

$$T = \left[1 + \frac{\left(e^{k_{II}a} - e^{-k_{II}a}\right)^2}{16\frac{E}{V_0}\left(1 - \frac{E}{V_0}\right)}\right]^{-1} = \left[1 + \frac{\sinh^2(k_{II}a)}{4\frac{E}{V_0}\left(1 - \frac{E}{V_0}\right)}\right]^{-1}$$

$$= \left[1 + \frac{\sinh^2(k_{II}a)}{4\frac{3}{10}\left(1 - \frac{3}{10}\right)}\right]^{-1} = \left[1 + \frac{100}{84}\sinh^2(k_{II}a)\right]^{-1}$$

$$T_p = \left[1 + \frac{100}{84}\sinh^2(5.80839\ldots)\right]^{-1} = 3.02936\ldots \times 10^{-5} \approx 3.0 \times 10^{-5}$$

$$T_d = \left[1 + \frac{100}{84}\sinh^2(8.21431\ldots)\right]^{-1} = 2.46409\ldots \times 10^{-7} \approx 2.5 \times 10^{-7}$$

The proton will be transmitted about 0.003% of the time, while the deuteron will be transmitted about 0.000025% of the time, and clearly $T_p > T_d$ showing that the proton is more likely to make it through the barrier.

4. (From Eisberg & Resnick, P 6-18, pg 229) A particle of total energy $9V_0$ is incident from the $-x$ axis on a potential given below. Find the probability that the particle will be transmitted on through to the positive side of the x axis, $x > a$. [10]

$$V(x) = \begin{cases} 8V_0, & x < 0, \\ 0, & 0 < x < a, \\ 5V_0, & a < x. \end{cases}$$

Solution: The potential energy function $V(x)$ with $E > V_0$ looks like figure 2.

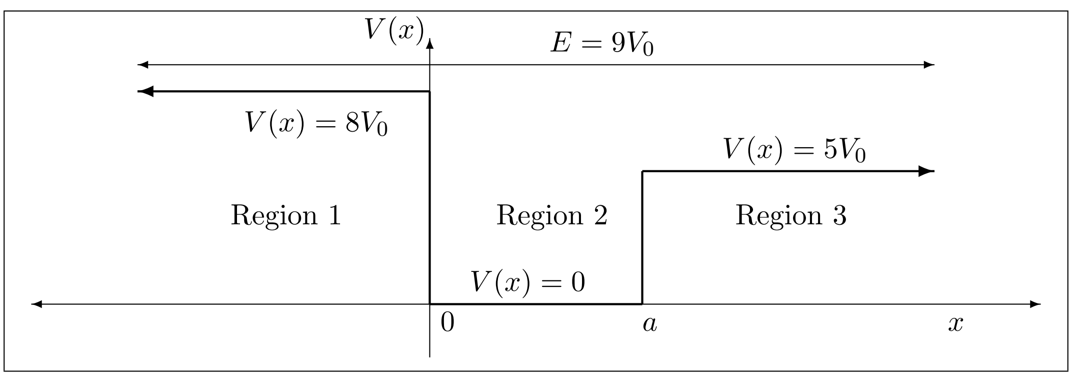

Figure 2: Un-symmetric well potential

We might be tempted to just use the T value found for a step function of the given height at $x = 0$ and multiply that by the T value for the step function of the given height at $x = a$ to give an overall T. In general, probabilities do multiply together to find the probability of two successive events, however for situations like this it is not a valid method. In effect, the reflected waves from the different boundaries create interference that makes the transmission coefficient a function of not only the potential heights, but also the thickness a.

Since $E > V(x)$ in all regions, the set up for this problem is identical with the set up for problem 2 above, up through (2.12). The only difference is in the values of k_1, k_2, and k_3, namely

$$
\begin{aligned}
k_1 &= \frac{\sqrt{2m(E - V(x))}}{\hbar} \\
&= \frac{\sqrt{2m(9V_0 - 8V_0)}}{\hbar} = \frac{\sqrt{2mV_0}}{\hbar} \\
k_2 &= \frac{\sqrt{2m(9V_0)}}{\hbar} = 3\frac{\sqrt{2mV_0}}{\hbar} = 3k_1 \\
k_3 &= \frac{\sqrt{2m(9V_0 - 5V_0)}}{\hbar} = 2\frac{\sqrt{2mV_0}}{\hbar} = 2k_1.
\end{aligned}
$$

Putting $k_2 = 3k_1$ and $k_3 = 2k_1$ into (2.12) gives us

$$
\begin{aligned}
T^{-1} &= \frac{1}{4k_1k_2^2k_3}\left[(k_1k_2 + k_2k_3)^2\cos^2(k_2a) + (k_1k_3 + k_2k_2)^2\sin^2(k_2a)\right] \\
&= \frac{1}{4k_19k_1^22k_1}\left[(k_13k_1 + 3k_12k_1)^2\cos^2(3k_1a) + (k_12k_1 + 3k_13k_1)^2\sin^2(3k_1a)\right] \\
&= \frac{1}{72}\left[81\cos^2(3k_1a) + 121\sin^2(3k_1a)\right] \\
&= \frac{81}{72}\left[\cos^2(3k_1a) + \frac{121}{81}\sin^2(3k_1a)\right] \\
&= \frac{81}{72}\left[1 - \sin^2(3k_1a) + \frac{121}{81}\sin^2(3k_1a)\right] \\
&= \frac{81}{72}\left[1 + \frac{40}{81}\sin^2(3k_1a)\right] = \frac{1}{72}\left[81 + 40\sin^2(3k_1a)\right] \\
T^{-1} &= \left[\frac{9}{8} + \frac{5}{9}\sin^2(3k_1a)\right]^{-1} \\
&= 72\left[81 + 40\sin^2(3k_1a)\right]^{-1}
\end{aligned}
$$

Thus, the probability of the particle being transmitted varies between a maximum of $8/9 \approx 0.88889$ (when a is such that $\sin(3k_1a) = 0$, namely $a = n\pi/(3k_1)$) and a minimum of $72/121 \approx 0.6$ (when a is such that $\sin^2(3k_1a) = 1$, namely $a = (n + 1/2)\pi/(3k_1)$).

5. (From Eisberg & Resnick, P 6-20, pg 230) Two possible eigenfunctions for a particle moving freely in a region of length a, but strictly confined to that region, are shown in Eisberg & Resnick, Figure 6-37, pg 230. When the particle is in the state corresponding to the eigenfunction ψ_{I}, its total energy is $4\,\mathrm{MeV}$.
   (a) What is its total energy in the state corresponding to ψ_{II}? [5]
   Solution: ψ_{I} has three nodes and two anti-nodes, so it is the first excited state, so $n = 2$. Since the energy of the state is proportional to n^2, the energy of the $n = 2$ state will be $2^2 = 4$ times the energy of the $n = 1$ state, so the energy of the $n = 1$ state must be $E_1 = 1\,\mathrm{MeV}$. ψ_{II} has four nodes and three anti-nodes, so it is the second excited state with $n = 3$. $E_{\mathrm{II}} = E_3 = 3^3E_1 = 9\,\mathrm{MeV}$.
   (b) What is the lowest possible total energy for the particle in this system? [5]
   Solution: As stated above, the lowest energy is the $n = 1$ state with $E_1 = 1\,\mathrm{MeV}$.

Headstart for next week, Week 12, starting Monday 2004/12/06:
– Read Chapter 6 "Solutions of Time-Independent Schroedinger Equation" in Eisberg & Resnick
– – Section 6.9 "The Simple Harmonic Oscillator Potential"
– – Section 6.10 "Summary"
– Review notes, review texts, review assignments, learn material, do well on exam