Why tenure?

Bruce Cater, Byron Lew and Marcus Pivato*

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Abstract

Academic research is a public good whose production is supported by the tuition-paying students that a faculty's research accomplishments attract. A professor's spot contribution to the university's revenues thus depends not on her spot research production, but rather on her cumulative research record. We show that a profit-maximizing university will apply a 'high' minimum retention standard to the production of a junior professor who has no record of past research, but a 'zero' retention standard to the spot production of a more senior professor whose background includes accomplishments sufficient to have cleared the 'high' probationary hurdle.

I Introduction

Under a tenure-track contract, a professor who fails to meet some positive standard of research production during a finite probationary period is dismissed at that period's end. Yet, a professor who meets that initial standard is granted *tenure* and retained *regardless* of her research output thereafter.[1]

In recent decades, economists have offered a number of explanations of this puzzling contractual choice. Freeman (1977) suggests that risk averse professors are granted the security of tenure to compensate for the risk inherent in their research.[2] Yet, this explanation is unsatisfactory, for nonacademic employers manage to contract with workers who

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*Cater and Lew: Department of Economics, Trent University, Ontario, Canada; Pivato: Department of Mathematics, Trent University, Ontario, Canada.

[1] Siow (1998) notes that, in the 1989 *Survey Among College and University Faculty* sponsored by the Carnegie Foundation Survey, 4.7 percent and 36.4 percent of tenured faculty in doctoral-granting and non-doctoral-granting schools, respectively, reported no publications in the previous two years and no current research. Yet, in reviews of U.S. case law, legal scholars, including Hendrickson (1988) and Morris (1992), do not cite a single case in which a tenured professor was dismissed primarily on the grounds of low research productivity.

[2] Kahn and Huberman (1988) and Waldman (1990) offer explanations of the use of 'up-or-out' contracts, but do not address the issue of post-probationary minimum production standards. McKenzie (1996) and McPherson and Shapiro (1999) attempt to explain academic tenure on internal political, rather than economic, grounds.

are risk averse and whose productivity is uncertain without having to offer them anything akin to tenure.

Carmichael (1988) suggests that a university is unique in that, because the state of academic knowledge is vast and expanding, it is the incumbent occupants of its scarce faculty slots who alone can judge the research potential of candidates. To maximize its research production, the university then provides those incumbents with the security of tenure to ensure that they are willing to identify and hire candidates superior to themselves. Yet, while clearly insightful, this theory is incomplete. Tenure-track contracts pre-specify the time at which a tenure decision is to be made. However, Carmichael's hiring incentives could be created by granting tenure on an *ad hoc* basis to an accomplished incumbent, when the need for her input into a hiring decision first arises. Moreover, a university grants tenure to *all* incumbents who clear its probationary hurdle. But in Carmichel's story, why couldn't a university increase its research production by granting tenure and hiring authority only to some small 'elite' fraction of its faculty, while leaving other post-probationary members subject to termination if their ongoing production is poor? Most importantly, in focusing on the issue of incentives in hiring, Carmichael's approach assumes that a professor's research production is governed only by ability, and not effort, and that ability is constant over her lifetime. In doing so, it totally fails to consider a question at the heart of the tenure debate: why does academic research output decline, on average, with age?[3] Does this pattern reflect some disincentive effect, and therefore a major drawback of the tolerance tenure extends? Or can declining production instead be understood to be optimal in some way?

Siow (1998) links tenure to declining research production in an ingenious way, arguing that, as research productivity falls with age, it becomes socially efficient for a professor to spend less time on research and more time on teaching. Tenure then solves an effort allocation problem by inducing older professors to do less research. But this theory is also unsatisfactory. If a university's goal were to *reduce* research effort among its older faculty, it might tolerate *reduced* production, but not utter failure. On the other hand, if its goal were to *eliminate* research effort among older faculty, then why does the university provide an elaborate system of research incentives and support to tenured professors, such as teaching reductions, prizes and awards, internal research grants, sabbaticals, and laboratory facilities?

The traditional case for government funding of academic research is well known: because the social value of certain kinds of ideas cannot be fully or even partially appropriated by their developers, the private sector will underinvest in this kind of work.[4] This traditional view does not, however, explain why this work occurs in universities, when the subsidies that support it could instead be directed to private corporations. Aghion et al (2008) offer an explanation. They argue that, because it is important to economize on the wage and monitoring costs of those who engage in research characterized by a low (expected) appropriable value, it is efficient to grant them research 'freedom', for they value creative

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[3] Evidence of this pattern is presented in Diamond (1986) and Levin and Stephan (1991).

[4] Although Nelson (1959) and Arrow (1962), in first describing this problem, focused explicitly on basic scientific research, their reasoning clearly applies to any field of research characterized by imperfect appropriability.

control and would have to be paid a premium to give it up.[5] Because it is efficient for research of this kind to be done by 'free' researchers, it follows that a university, where researchers are granted the freedom of tenure, is the place where work of this kind should be done.[6] As an explanation of academic tenure, however, this theory seems somehow incomplete. In taking research 'freedom' to be an exogenous characteristic of university employment, it ignores the fact that, just as subsidized research could take place in private firms, research 'freedom' could be granted to the researchers those firms employ. How, then, are we to understand why research characterized by little or no appropriable value is heavily concentrated in universities, and why only universities grant tenure? Is there some efficiency that a university alone can realize?

In this paper, we develop a model that explains why a university would retain only those professors who are initially successful in research, regardless of their research output thereafter, and why it would then continue to induce research effort, albeit at a rate that declines with the professor's age. Our model is based on what we take to be the unique, defining characteristic of academic productivity.

Specifically, for potential students who find it difficult to evaluate a faculty's knowledge directly, observable research accomplishments may be a reliable proxy, either because those who conduct research will accumulate knowledge or because those who are more knowledgeable will find it less costly to do research. In the spirit of Bok (1986), James (1990) and Hearn (1992), we thus argue that a professor's spot contribution to a university's tuition revenues will depend on the strength of her cumulative research record.[7] This assumption is supported by the empirical work of Siow (1997), who finds that schools with more successful researchers have larger shares of out-of-state and foreign students.

Thus, the advantage of concentrating research with little or no appropriable value in universities, rather than in private firms, may be that, because the work will be effectively 'subsidized' by the tuition-paying students it attracts, the burden on governments to directly subsidize it is reduced. But out of the university's unusual revenue structure comes some unusual profit-maximizing behavior. It turns out that the particular tolerance of tenure can be understood as simply the means by which the university retains those professors whose current research production may be poor or nonexistent, but whose past research accomplishments continue to make them profitable through the tuition revenues they attract. Moreover, declining research production can be understood as resulting not from some disincentive effect of tenure, but rather from the university optimally inducing less research effort as a professor approaches retirement and the opportunities to realize tuition revenues from any resulting research successes diminish.

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[5] Note that, while Aghion et al's focus is on 'potentially' commercializable research (e.g., biotechnology), their logic regarding the efficiency of economizing on wages would also seem to apply to research in fields such as pure mathematics, public policy and philosophy, where research yields *no* appropriable results.

[6] Strictly speaking, Aghion, Dewatripont and Stein's (2008) definition of 'academic freedom' is the freedom to produce *any* research. The feature of tenure we seek to explain here is the 'freedom' to produce *no* research. Note, however, that it is impossible for a university to credibly guarantee 'academic freedom' in the sense of Aghion et al without de facto providing 'tenure' in the sense we consider here.

[7] Of course, students need not observe and process the research directly for the signal to be effective. Scholarly accomplishments of a university's faculty may filter down to students through media sources that rank universities, in part, on the basis of those accomplishments and their correlates.

This paper is related to the work of Cater, Lew and Smith (2008), who examine a simpler model in which research productivity is governed only by ability, not effort, and in which that ability is assumed to decline with age. The university's problem in that paper thus involves choosing only the conditions under which an incumbent will be retained into the next period. The main contribution of this paper is our consideration of a much more general model that allows for the simultaneous analysis of the university's choices of optimal research standards and research effort inducement. The analysis presented here enables us to fully resolve the contractual puzzle described above.

II The model

A representative university

A government or private donor provides a one-time capital endowment to create a representative university under the terms of a charter that directs it, in perpetuity, to produce and impart academic knowledge. The endowed capital is sufficient to support a fixed number of faculty 'slots'. Without loss of generality, we let that number be one.[8]

Once endowed, the university is expected to be financially independent. It has a zero rate of discount. Operating in discrete time, it expects to live for infinitely many periods, remaining viable by maximizing its expected profits per period.[9]

The social value of any research its faculty produces cannot be appropriated by the university.[10] Moreover, direct research subsidies (i.e., grants), if and when they are received, merely cover explicit research costs (e.g., materials and laboratory equipment), and thus have no net effect on the university's profits.

The university's only revenues are 'tuition' fees, defined here to include any revenues tied to student enrollment.[11] Its only costs are the wages of its faculty.

Hiring and retention decisions can be made only at the beginning of a period. The university can condition a professor's wages on her observable research output. Its employment contracts are enforceable before the courts.

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[8] In a competitive instruction market, the endowed capital effectively creates a capacity constraint for each university, for while a particular university in our model could, say, mortgage additional capital to expand its capacity to research and teach, doing so would put it at a cost disadvantage relative to other schools who operate strictly on the basis of their endowed capital.

[9] Rothschild and White (1995) and Siow (1998) also assume that a university seeks to maximize profits.

[10] We abstract from the fact that some research discoveries *can* be commercialized to focus on the questions of why tenure is granted in academic fields, such as philosophy, public policy and pure mathematics, where research yields *no* appropriable results, and why, historically, tenure arose decades before the Bayh-Dole Act of 1980 first gave academic institutions the right to patent and commercialize the results of government-subsidized research.

[11] Our model thus applies both to privately-endowed schools where tuitions are typically paid entirely by the students, and to publicly-endowed universities where tuitions may be subsidized, in part or in full, by a government.

A representative professor

There is an infinite pool of potential professors, drawn from overlapping generations, each with a working lifetime of three periods. Those in their first, second and third periods will be referred to, respectively, as being 'junior', 'middle-aged' and 'senior'.

In each period of her working life, a representative, potential professor will occupy either a nonacademic or an academic job. The option of nonacademic employment always exists; its per period maximized utility is a constant C_o that sets a floor on the utility that academic employment must provide. If a professor chooses the nonacademic option at the beginning of any period, we, like Carmichael (1988), assume that her academic abilities decay so that, in any subsequent period, nonacademic employment will be her only option.[12]

At the beginning of her first working period, our representative potential professor receives one offer of academic employment. If she accepts that offer, then, at the beginning of the second period, the university that employed her as a 'junior' may wish to retain her. Outside universities may also attempt to hire her, and a bidding war for her services may occur. A similar process then occurs at the beginning of the third period if she remains in academic employment through her second working period.[13]

If employed by a university during the t^{th} period of her working life ($t = 1, 2, 3$), our representative professor will, at the beginning of that period, choose an unobservable level of research effort, e_t (≥ 0), the quadratic utility cost of which is e_t^2. At the end of the period she will then realize research output described by a single index that, as in Carmichael (1988), measures quantity and quality with the correct weights. The value of that index, r_t, is drawn randomly from the probability distribution ρ_{e_t} on $[0, \infty)$. We will consider three different families of distributions: the uniform, the exponential, and the power-law.[14] As in Carmichael (1988), any knowledge accumulated through, or otherwise associated with, academic research is of no value in nonacademic employment. During any period of academic employment, our representative professor will also provide instruction, the disutility of which is a constant D. We normalize the professor's utility scale so that $C_o + D = 0$.

Our key assumption is that a professor's period t research output serves as a signal of knowledge that increases her contribution to the tuition revenues of *any* university that employs her in *any* subsequent period. Because all 'junior' professors in our model begin with no research record, they all contribute the same revenues during the first period of their working lives. We normalize those revenues to 0. We assume research output is measured in terms of the dollar-value of tuition revenues it indirectly generates for the university (rather than in, say, quality-adjusted pages of peer-reviewed publications, or quality-adjusted citation counts, or some other measure). Thus, we suppose there are

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[12] This simplifying assumption rules out the possibilities of delayed or discontinuous academic employment.

[13] Note that, in our model, because a professor's research record is publicly observable, there is no meaningful distinction between an outside university raiding for a professor and a professor seeking employment with an outside university. It is, therefore, sufficient to consider only the implications of raiding.

[14] These three probability distributions, each intuitively plausible and analytically tractable, are chosen to demonstrate that our results are robust across a range of models of intellectual creativity.

constants $k_1 > 0$ and $k_2 > 0$ such that a middle-aged professor contributes $k_1 r_1$ dollars of revenue, while a senior professor contributes $k_2 r_1 + k_1 r_2$, where r_1 and r_2 are period-1 and period-2 research respectively (the ratio k_2/k_1 measures the relative marginal revenues of older research versus more recent research; for simplicity we will generally assume $k_1 = k_2$). Note that r_3 generates no tuition revenue for the university: the professor's realization of r_3 coincides with her retirement, so the knowledge represented by this research leaves with her, and therefore cannot enhance the reputation of the university.[15]

Our representative professor is risk neutral and has a zero rate of discount. Her constant marginal utility of money is normalized to 1. In choosing between alternative employment offers and levels of effort, she will, therefore, attempt to maximize her expected lifetime income, less any research effort disutility.

At the beginning of the first period of her working life, our representative professor is assumed to accept the academic offer, provided that it matches or betters the expected lifetime utility of 0 she would obtain from a lifetime of nonacademic employment. Because of infinitesimally small but positive job change costs, an academic job which offers an expected future utility of 0 is similarly sufficient to deter a 'middle-aged' or 'senior' professor from quitting to pursue nonacademic employment. Those job change costs also mean that, for one university to successfully raid another university for a 'middle-aged' or 'senior' professor, the recruiting university must slightly better the (expected) wage she would receive by remaining with her current employer.

III Analysis

Academic contracts

Any equilibrium in our model necessarily involves at least some universities hiring 'junior' professors at least some of the time. The terms of employment offered to a 'junior' will not only determine whether she accepts the initial academic offer, but will also play a role in determining the relative value of her nonacademic and potential academic options in subsequent periods. It is, therefore, necessary for us to first describe the terms of employment that a 'junior' professor will be offered.

When attempting to hire a 'junior', a university must choose two inter-related features of its employment contract: (1) the conditions, if any, under which it wishes to retain the professor into subsequent periods of her working life and (2) the wage structure necessary to recruit her initially, to induce her 'optimal' effort, and to ensure that she chooses to remain with the university when her retention is sought.

In each period, we assume that the university sets a minimum standard that the professor's most recent research realization must equal or exceed for her to be given the option of remaining. This structure admits the tenure-track sequence of 'spot' standards as a possible (partial) solution to the university's problem, but in no way restricts the values

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[15] There may be rare cases where a university continues to realize revenues from its association with a particularly accomplished professor even after her retirement. We abstract from this possibility, however, on the grounds that the use of tenure-track contracts seems to transcend such cases.

of those standards. In each period, the contract pays a base wage, plus a bonus that is linear in research output.[16]

The *academic contract* offered to a 'junior' professor is thus a structure, $\mathbf{C} := (w_1, w_2, w_3; b_1, b_2; b_{21}, b_{31}, b_{32}; s_1, s_2)$, comprised of base wages (w_1, w_2, w_3), bonus multipliers $(b_1, b_2; b_{21}, b_{31}, b_{32})$, and retention standards (s_1, s_2).

A professor who accepts $\mathbf{C}$ will receive a salary of

$$S_1(r_1) \quad := \quad w_1 + b_1 r_1 \tag{1}$$

at the end of her first period of employment. In the event that her first research draw $r_1 \geq s_1$, she then has the option of remaining with the university through her second period. If she chooses to remain, she receives a salary of

$$S_2(r_1, r_2) \quad := \quad w_2 + b_2 r_2 + b_{21} r_1 \tag{2}$$

at the end of that period. Similarly, if her $r_2 \geq s_2$, she is given the option of remaining with the university through the third and final period of her working life. If she takes that option, she receives a salary of

$$S_3(r_1, r_2) \quad := \quad w_3 + b_{31} r_1 + b_{32} r_2. \tag{3}$$

at that period's end. Note that the contract contains no bonus for r_3. Because the professor's working life ends immediately after any r_3 draw, that draw can result in no additional revenues for the university, making it obvious that payment of a bonus for that draw will never be profitable. Note also that our contract's general payment structure places no restrictions on the timing of research bonuses, allowing them to be paid, if at all, immediately upon the research realization and/or in any subsequent period of retention.

A number of definitions are useful. For any level of research effort, e (≥ 0), let $\overline{r}(e) := \int_0^\infty r \; d\rho_e[r]$ be the expected value of r. For any $s \geq 0$, let

$$P(e, s) \; := \; \int_s^\infty d\rho_e[r] \quad \text{and} \quad \overline{R}(e, s) \; := \; \frac{1}{P(e, s)} \int_s^\infty r \; d\rho_e[r] \tag{4}$$

be, respectively, the probability that $r \geq s$, and the expected value of r, given that $r \geq s$.

We define the *net benefit* for the professor during period t to be the net benefit of remaining employed by *this* university under contract $\mathbf{C}$, rather than quitting to nonacademic employment. (We will deal with the possibility of quitting to another university later.) The expected net benefit she extracts from $\mathbf{C}$ depends upon the effort she exerts.

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[16] We assume this 'linear' incentive structure to obtain a tractable model. Macleod and Malcomson (1989), Pearce and Stacchetti (1998), and Hogan (2002) consider similar payment schemes. In those models, only the base wage is part of the explicit contract; the bonus for unobservable effort is promised only 'implicitly', but, in repeated interaction, it is in the best interest of the firm to honor even the implicit component. In our model, the bonus is tied to observable research output, so both the base wage and bonus components are explicit. In a very general principle-agent model, Holmstrom and Milgron (1987) have shown the optimality of a linear compensation scheme based on aggregate performance. Strictly speaking, their result (which assumes a Gaussian distribution for the agent's output, and no possibility of termination) is not applicable to our model (which assumes non-Gaussian distributions and does allow for termination); however, it makes our linear incentive structure a plausible simplifying assumption.

A professor's incentives

To maximize her lifetime expected net benefit under **C**, the professor must choose optimal effort levels for each period. To do this, she solves a dynamic programming problem, starting with period 3 of her working life and working backward.

Period 3. – Suppose the professor has been retained under **C** through the first two periods of her career, and that her $r_2 \geq s_2$. If she were to remain with the university, then, in the absence of any bonus for third period research production, her optimal e_3 would be 0, so the net benefit of remaining with this university for the third period of her career would be

$$\mathrm{NB}_3(r_1, r_2) \quad \underset{(3)}{=} \quad w_3 + b_{31} r_1 + b_{32} r_2. \tag{5}$$

To ensure that the professor would not instead choose to pursue nonacademic employment, we require:

$$\mathrm{NB}_3(r_1, r_2) \quad \geq \quad 0, \quad \forall\ r_1, r_2 \geq 0. \tag{6}$$

Period 2. – Now suppose that a professor has completed the first period of **C**, that her $r_1 \geq s_1$, and that (6) is satisfied. The expected net benefit of choosing to remain with the university for (at least) the second period of her working life would be

$$\mathrm{NB}_{2,3}(r_1, e_2) \quad = \quad \mathrm{NB}_2(r_1, e_2) + P(e_2, s_2)\, \mathrm{NB}_3(r_1, e_2). \tag{7}$$

Here,

$$\mathrm{NB}_2(r_1, e_2) \quad := \quad w_2 + b_2 \overline{r}(e_2) + b_{21} r_1 - e_2^2 \tag{8}$$

is the net benefit of period 2 employment alone, while

$$\mathrm{NB}_3(r_1, e_2) \quad := \quad w_3 + b_{31} r_1 + b_{32} \overline{R}(e_2, s_2).$$

is the expected value of (5), given period-2 effort e_2. If the professor were to choose to remain with the university, she would then choose her optimal level of period-2 research effort, e_2^*, so as to maximize (7). For the university to ensure that a professor will not pursue her nonacademic option at this stage, the contract must satisfy

$$\mathrm{NB}_{2,3}(r_1, e_2^*) \quad \geq \quad 0, \quad \forall\ r_1 \geq 0. \tag{9}$$

Period 1. – Suppose (6) and (9) are satisfied. For a 'junior' professor, aware that her period-2 research effort will be e_2^*, the expected net benefit of the academic contract, given period-1 effort e_1, is

$$\mathrm{NB}_{1,2,3}(e_1, e_2^*) \quad = \quad \mathrm{NB}_1(e_1) + P(e_1, s_1)\, \mathrm{NB}_{2,3}(e_1, e_2^*). \tag{10}$$

Here,

$$\mathrm{NB}_1(e_1) \quad := \quad w_1 + b_1 \overline{r}(e_1) - e_1^2 \tag{11}$$

is the net benefit of period 1 alone, while

$$\mathrm{NB}_{2,3}(e_1, e_2^*) \;:=\; w_2 + b_2 \overline{r}(e_2^*) + b_{21} \overline{R}(e_1, s_1) - (e_2^*)^2 + P(e_2^*, s_2) \left(w_3 + b_{31} \overline{R}(e_1, s_1) + b_{32} \overline{R}(e_2^*, s_2) \right)$$

is the expected value of (7), given period-1 effort e_1 and anticipating optimal period-2 effort e_2^*. If the 'junior' professor were to accept the academic contract, she would choose her optimal level of period-1 research effort, e_1^*, so as to maximize (10). It will be rational for the potential 'junior' professor to accept the academic offer if and only if $\mathrm{NB}_{1,2,3}(e_1^*, e_2^*) \geq 0$.

To the university, w_1 represents a cost that has no influence on the professor's choice of effort profile, (e_1^*, e_2^*). To minimize its costs, the university will set $w_1 := -b_1\overline{r}(e_1^*) + (e_1^*)^2 - P(e_1^*, s_1)\,\mathrm{NB}_{2,3}(e_1^*, e_2^*)$, so that the contract satisfies the *minimal recruitment* condition:

$$\mathrm{NB}_{1,2,3}(e_1^*, e_2^*) \quad = \quad 0. \tag{12}$$

We will say that $\mathbf{C}$ is *admissible* if it satisfies (6), (9) and (12). Period-specific expected profits, as of the beginning of each of the contract's three periods, are then given by:

$$\begin{aligned} \overline{\Pi}_1 &= -w_1 - b_1\overline{r}(e_1^*), & (13)\\ \overline{\Pi}_2(r_1) &= -w_2 - b_2\overline{r}(e_2^*) + (k_1 - b_{21})r_1 \qquad \text{and} & (14)\\ \overline{\Pi}_3(r_1, r_2) &= -w_3 + (k_2 - b_{31})r_1 + (k_1 - b_{32})r_2. & (15) \end{aligned}$$

Academic raiding

Lazear (1986), Bernhardt and Scoones (1993) and Waldman (1990) each describe a situation where one firm initially employs a worker who might be raided by an outside firm. Each of those papers establish that raiding will occur only if the worker is a better match with the outside firm; where match-quality is equal across the firms, the initial employer creates a contract to pre-empt raiding. In our model, there is no match-quality heterogeneity. A professor has no preference for one university over another, and, conditional on her research record, she would generate the same revenues for any university that employs her; thus, a university that hires a 'junior' professor need also consider the possibility that it could be raided for its 'middle-aged' and/or 'senior' professors by another university.

We say that the contract $\mathbf{C}$ is *raid-proof* if, for all $r_1, r_2 \geq 0$, we have $\overline{\Pi}_2(r_1) \leq \overline{\Pi}_1$ and $\overline{\Pi}_3(r_1, r_2) \leq \overline{\Pi}_1$, where these quantities are as defined in equations (13-15). Consider an economy involving a large number of universities competing to employ a large number of professors. In a competitive equilibrium in such an economy, every university must adopt a raid-proof contract. To see this, suppose that University X adopted a contract that was not raid-proof. Then at least some professors at X will achieve research realizations r_1 or r_2 such that $\overline{\Pi}_2(r_1) > \overline{\Pi}_1$ or $\overline{\Pi}_3(r_1, r_2) > \overline{\Pi}_1$ —let's call these professors the *stars*. Then another ('raider') University Y could adopt the following employment strategy: Y hires no junior professors of its own, but instead simply 'raids' all the stars from X by offering them slightly higher salaries (in effect, offering a contract with slightly higher values of b_{21}, b_{31} and/or b_{32}). University X loses all its stars to University Y, so all the professors remaining at X generate expected per-period profits of $\overline{\Pi}_1$ or less. Thus, the expected per-period profits of X as a whole are at most $\overline{\Pi}_1$. On the other hand, University Y, having hired nothing but the stars, earns expected per-period profits strictly *greater* than $\overline{\Pi}_1$. Clearly, this scenario is not profit-maximizing for University X.

We can thus assume that our representative university operates in a *raid-proof equilibrium*, where it and all the other universities deploy raid-proof contracts.

The optimal contract

We say that a contract is *tenure-track* if $s_1 > 0$ and $s_2 = 0$ (or, equivalently, $0 < P(e_1, s_1) < 1$ and $P(e_2, s_2) = 1$ for any $e_1, e_2 \geq 0$). We say that the contract induces a *declining effort profile* if $e_1^* > e_2^*$. We now come to our main result.

Theorem 1 *Suppose $k_1 = k_2$ and assume a raid-proof environment. Let $\{\rho_e\}_{e\in\mathbb{R}_{\not-}}$ be a family of probability distributions on $[0, \infty)$, and let $\mathbf{C}$ be an admissible, raid-proof contract that maximizes expected profits per period.*

- **(a)** *For all $e \geq 0$, suppose ρ_e is the uniform probability distribution on $[0, e]$.* (That is, $d\rho_e(r) = 1/e$ if $r \in [0, e]$ and $d\rho_e(r) = 0$ if $r > e$.) *Then $\mathbf{C}$ is tenure-track, with a declining effort profile.*
- **(b)** *For all $e \geq 0$, suppose ρ_e is the exponential probability distribution $d\rho_e(r) = \frac{1}{e}\exp(-r/e)$. Then $\mathbf{C}$ is tenure-track, with a declining effort profile.*
- **(c)** *For any $\alpha > 1$ and $e \geq 0$, let ρ_e^α be the power law distribution $d\rho_e^\alpha(r) = \frac{e^\alpha\,\alpha}{(e+x)^{\alpha+1}}$. There exist $\underline{\alpha}, \overline{\alpha} \in (1, \infty)$ such that, if $\alpha \in (1, \underline{\alpha})$ or $\alpha \in (\overline{\alpha}, \infty)$, then $\mathbf{C}$ is tenure-track, with a declining effort profile. In particular, this holds if $\alpha = 2$.*

In Theorem 1, we see that the maximization of profits leads the university both to adopt a tenure-track contract and to induce declining effort that results in research production declining, on average, over the life cycle.[17]

The intuition is straightforward. In each of the first and second periods of the contract, the university will apply the same rule: induce a professor's research effort up to the point where the resulting marginal revenue product is equal to her (increasing) marginal cost. But because more revenues can be realized from the first research draw than from the second, the optimal level of induced research effort declines from the first to the second period. Accordingly, research output will, on average, decline with age.

In choosing the retention standard to apply at the beginning of each of the second and third periods, the university faces a trade-off: increasing the minimum standard applied to the most recent research draw raises the conditional mean payoff associated with the remaining periods of the contract, but it involves foregoing any benefit from the past research accomplishments of those who fail to meet the current standard. For a professor entering the second period of her working life, whose first research draw will influence both second and third period profitability and who has no record of past accomplishments, a relatively high minimum standard is optimal. But for a professor entering her third period, whose second research draw will influence only third period profitability and whose background includes the accomplishments necessary to have cleared the high first standard, the university optimally tolerates little or even no research output on the professor's part.[18]

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[17] As a rather technical point, we note that, while we have assumed that $k_2 = k_1$ in order to simplify the proof, we do not believe this assumption is critical to our conclusions. In particular, the same result should hold when $k_2 > k_1$ (because this would simply *increase* the value of a senior professor relative to her junior potential replacement), and may even hold for $k_2 < k_1$ (as long as k_2/k_1 is close enough to 1).

[18] Because, in our model, inducing research contributes to revenues only *ex post*, and only on the condition that the worker is retained, it can be thought of as being analogous to a nonacademic employer's investment

Our model predicts that a young professor will be hired under a contract characterized by the tenure-track sequence of firing standards. Moreover, because the realization of profits is tied to a professor developing a research record that would increase her value to *any* university, we have argued that the tenure-track contract our representative university offers will be further characterized by a back-loaded wage structure that prevents it from being raided for those professors it wishes to retain following the probationary period.

A professor in our model will thus experience one of two career paths: after being hired under a tenure-track contract by a particular university, she will either be granted tenure and remain with that same university for the balance of her career or be denied tenure then make a permanent transition into nonacademic work.

Yet, in practice, career paths may deviate from those that our model predicts in two ways. First, while universities hire most faculty on a tenure-track, they also employ some faculty on a limited-term basis. Second, while university faculty report much less mobility than workers in general, some movement between academic jobs does occur. Barbezat and Hughes (2001) find that, while 36 percent of university faculty report having held only one academic job in their careers, 32 percent report having held two jobs and a further 19 percent report having held three academic jobs.

How can these realities be reconciled with our result? Our assumption regarding the translation of a professor's current research output into future tuition revenues is itself based on the implicit assumption that there will exist future demand for the instruction each faculty slot provides. Under these conditions, we have shown that any slot will be optimally filled with a tenure-track hire. But if the university were uncertain as to whether some portion of its current demand would continue into the future, a limited-term hire could be used to meet that portion of current demand without any long-term commitment.

In this light, a broader range of academic career paths can be understood. A professor who reports having held one academic job is likely tenured or on a tenure-track. A professor who has held two or three academic jobs since graduation, on the other hand, may have held one or two post-doctoral or limited-term positions prior to landing a tenure-track job, or they may have simply worked for two or three different universities on a limited-term basis. These alternative paths involving early-career movement seem loosely consistent with further empirical evidence that, during their thirties, academics acquired an average of 1.8 new jobs per capita (one of which was likely a first position following their doctoral studies), while professors in their forties and fifties experienced 0.29 and 0.26 job-changes per capita, respectively (Barbezat and Hughes, 2001).[19] Of course, some of the observed mobility surely represents voluntary movement between tenure-track or tenured jobs. But because movement from one academic job to another is associated with, if anything, a small salary reduction (Barbezat and Hughes, 2001), even these transitions are consistent

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in training a worker. Indeed, our story can be thought of as being akin to one where the optimal rate of the training investment diminishes as the worker approaches retirement, and where the employer will not tolerate a worker who fails in his initial training, but *will* tolerate a worker who initially reaches an acceptable level of productivity, even if his productivity then ceases to increase any further.

[19] These statistics lie in sharp contrast to the results presented in Polachek and Siebert (1993), who show that, overall, U.S. men average 3.1 new jobs in their twenties, 2.1 new jobs in their thirties, 1.4 new jobs in their forties, and 0.9 new jobs in their fifties.

with our raid-proof equilibrium.[20] Such mobility may result from preference shifts that are beyond the scope of our model.

IV Outline of Theorem 1 proof

The proof of Theorem 1 is long and appears in the appendix. This section, however, describes the basis for that proof and outlines the major steps involved. (Detailed proofs of all statements appear in the Appendix.)

Recall that our representative university operates in a raid-proof equilibrium. In any period, the university will find itself in one of three 'states': its single faculty 'slot' will be occupied by a 'junior' professor (state 1), a 'middle-aged' professor (state 2), or a 'senior' professor (state 3). Whenever a 'junior' ('middle-aged') incumbent is retained into the following period, the university will transition from state 1 (2) to state 2 (3). If the university cannot 'raid' from other universities, then it can only hire junior professors; thus, whenever any incumbent is not retained into the following period, the university returns to state 1. If other universities will not 'raid' from our representative university, then the probability of retaining a professor is exactly the probability that her research exceeds the minimum standards s_1 and s_2 specified by the contract. Thus, the retention probabilities are $p_1 := P(e_1^*, s_1)$ and $p_2 := P(e_2^*, s_2)$. This data defines a 3-state Markov process with transition probability matrix

$$\left[\begin{array}{ccc} 1-p_1 & p_1 & 0 \\ 1-p_2 & 0 & p_2 \\ 1 & 0 & 0 \end{array}\right]. \tag{16}$$

This process has stationary probability distribution (π_1, π_2, π_3) given by

$$\pi_1 \;=\; \frac{1}{1+p_1+p_1p_2}, \quad \pi_2 \;=\; \frac{p_1}{1+p_1+p_1p_2}, \quad \text{and} \quad \pi_3 \;=\; \frac{p_1p_2}{1+p_1+p_1p_2}. \tag{17}$$

Recall that equation (14) gave the expected value of $\overline{\Pi}_2$ at the start of period 2 —i.e. once the realization of r_1 is already *known*. Likewise, (15) gave the expected value of $\overline{\Pi}_3$ at the start of period 3, when the realizations of r_1 and r_2 are both known. However, at the start of period 1, the future values of r_1 and r_2 are both unknown; at this moment, assuming $k_1 = k_2 = k$, the expected profits which **C** will generate in each of three periods of a professor's career are

$$\begin{array}{rcl} \overline{\Pi}_1 & \underset{(13)}{=\!=} & -w_1 - b_1\overline{r}(e_1^*); \\ \overline{\Pi}_2 & \underset{(14)}{=\!=} & -w_2 - b_2\overline{r}(e_2^*) + (k-b_{21})\overline{R}(e_1^*, s_1); \\ \text{and } \overline{\Pi}_3 & \underset{(15)}{=\!=} & -w_3 + (k-b_{31})\overline{R}(e_1^*, s_1) + (k-b_{32})\overline{R}(e_2^*, s_2). \end{array} \tag{18}$$

Combining (18) and (17), the expected profit per period of the university is given by

$$\overline{\Pi}(\,\mathbf{C}) \quad := \quad \pi_1\overline{\Pi}_1 + \pi_2\overline{\Pi}_2 + \pi_3\overline{\Pi}_3. \tag{19}$$

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[20] These wage dynamics are also noteably different from the nonacademic sector. Topel and Ward (1992), for example, found that job changes account for as much as one-third of wage growth among young men.

The university must find the (raid-proof) contract which maximizes the value of $\overline{\Pi}$. The proof of Theorem 1 now proceeds in three steps:

1. We relax the need to optimize over raid-proof contracts, by showing that a non-raidproof contract can be 'retroactively raidproofed' without affecting its optimality.

2. We show that it suffices to solve the optimization problem over a particularly nice class of contracts we call MNQ ('marginal no-quitting').

3. We establish Theorem 1 for the class of MNQ contracts.

Steps 1 and 2 both use the concept of *contract equivalence*. Let $\mathbf{C}$ and $\widetilde{\mathbf{C}}$ be two academic contracts. We say that $\mathbf{C}$ and $\widetilde{\mathbf{C}}$ are ***equivalent*** if:

(Eq1) In both contracts, the professor's optimal effort profile (e_1^*, e_2^*) is the same.

(Eq2) Both contracts have the same research standards (s_1, s_2).

(Eq3) Both contracts yield the same expected lifetime net benefit $\mathrm{NB}_{1,2,3}$ for the professor.

In particular, (Eq2) implies that $\mathbf{C}$ is tenure-track if and only if $\widetilde{\mathbf{C}}$ is also tenure-track. (Eq3) implies that $\mathbf{C}$ satisfies minimal recruitment condition (12) if and only if $\widetilde{\mathbf{C}}$ does.

Lemma 2 *If contracts* $\mathbf{C}$ *and* $\widetilde{\mathbf{C}}$ *are equivalent, then both contracts yield the same value of* $\overline{\Pi}$ *in equation* (19). (Thus, $\mathbf{C}$ is $\overline{\Pi}$-maximizing if and only if $\widetilde{\mathbf{C}}$ is.) □

The next proposition accomplishes Step 1 in our proof strategy. Recall that $\overline{r}(e) := \int_0^\infty r \, d\rho_e[r]$.

Proposition 3 *Assume* $\overline{r}'(e) \neq 0$ *for all* $e \geq 0$. *Let* $\mathbf{C}$ *be any admissible, tenure-track contract which is not raid-proof. There exists an admissible, raid-proof contract* $\widetilde{\mathbf{C}}$ *which is equivalent to* $\mathbf{C}$ (and hence, is also tenure-track). □

Proposition 3 says that, to demonstrate that the raid-proof $\overline{\Pi}$-maximizing contract is tenure-track, it suffices to first find a *non*-raid-proof contract which maximizes $\overline{\Pi}$ by being tenure-track, because we can always 'retroactively raidproof' it later.

We will focus on a class of contracts which are especially easy to optimize. We say that $\mathbf{C}$ is a ***minimal no-quitting*** (MNQ) contract if the conditions (6) and (9) are satisfied with *equalities* —that is,

$$\mathrm{NB}_{2,3}(r_1, e_2^*) = 0, \quad \text{and} \quad \mathrm{NB}_3(r_1, r_2) = 0, \quad \forall\ r_1, r_2 \geq 0. \tag{MNQ}$$

If $\mathbf{C}$ satisfies (MNQ), then $\mathrm{NB}_{2,3} = \mathrm{NB}_2$ and $\mathrm{NB}_{1,2,3} = \mathrm{NB}_1$; this will make it much easier to characterize (and control) the professor's utility-maximizing effort profile (e_1^*, e_2^*). Define $\beta : (0,\infty)\longrightarrow(0,\infty)$ by $\beta(e) := 2e/\overline{r}'(e)$ for all $e > 0$. We will require the family of distributions $\{\rho_e\}_{e\in\mathbb{R}_{\not-}}$ to satisfy the following assumption:

$$\beta \textit{ is a bijection from } (0,\infty) \textit{ to } (0,\infty). \tag{B}$$

One way to satisfy (B) is for β to be strictly increasing, with $\lim\limits_{e\searrow 0} \beta(e) = 0$, and $\lim\limits_{e\nearrow\infty} \beta(e) = \infty$. This just means that there are *not* strongly increasing returns to effort —a very weak assumption. It is easy to check that all the distribution families in Theorem 1 satisfy (B). The next proposition accomplishes Step 2 in our strategy.

Proposition 4 *Suppose* $\{\rho_e\}_{e\in\mathbb{R}_{\not -}}$ *satisfies* (B).

(a) *Let* $\mathbf{C}$ *be any contract satisfying minimal recruitment condition* (12). *There is a MNQ contract* $\widetilde{\mathbf{C}}$ *equivalent to* $\mathbf{C}$.

(b) *Let* $\mathbf{C}$ *be a profit-maximizing contract in the space of all admissible contracts. Let* $\widetilde{\mathbf{C}}$ *be a profit-maximizing contract in the space of all admissible MNQ contracts. Then* $\widetilde{\mathbf{C}}$ *provides the same expected profit per period as* $\mathbf{C}$. □

If hypothesis (B) holds, then Proposition 4(b) implies that, to find the $\overline{\Pi}$-maximizing contract, it suffices to maximize $\overline{\Pi}$ over the set of admissible MNQ contracts. For any MNQ contract, it can be shown that $b_{12} = b_{13} = b_{23} = w_3 = 0$, while the values of w_1 and w_2 are entirely determined by b_1 and b_2 (see Lemma A in the Appendix). Thus, an MNQ contract has only four free parameters: b_1, b_2, s_1, and s_2. Furthermore, we can achieve any desired effort profile (e_1, e_2) and retention probabilities (p_1, p_2) with a suitable choice of parameters $(b_1, b_2; s_1, s_2)$ (see Lemma B in the Appendix). Thus, the space of MNQ contracts can be parameterized by the set of all 4-tuples $(e_1, e_2; p_1, p_2)$. When an MNQ contract is expressed in this form, $\overline{\Pi}$ can be expressed as a function $\overline{\Pi}(e_1, e_2; p_1, p_2)$. With a mild technical assumption, we can then define functions $e_1^* : [0,1]^2 \longrightarrow \mathbb{R}_{\not -}$ and $e_2^* : [0,1]^2 \longrightarrow \mathbb{R}_{\not -}$ such that, for any fixed (p_1, p_2), the values of the parameters (e_1, e_2) which maximize $\overline{\Pi}(e_1, e_2; p_1, p_2)$ are $e_1^*(p_1, p_2)$ and $e_2^*(p_2)$ (see Lemma C). At this point, the $\overline{\Pi}$-maximization problem is reduced to finding the values of p_1^* and p_2^* in $[0,1]$ which maximize the function $\hat{\Pi}(p_1, p_2) := \overline{\Pi}[e_1^*(p_1, p_2), e_2^*(p_1, p_2); p_1, p_2]$. If the family of probability distributions $\{\rho_e\}_{e\in\mathbb{R}_{\not -}}$ and the derivative $\partial_2 \hat{\Pi}$ satisfy certain technical conditions, then the $\hat{\Pi}$-maximizing value of p_2 is $p_2^* = 1$ —in other words, the $\overline{\Pi}$-maximizing MNQ contract is tenure track (see Lemma E(a)). Furthermore, if p_1^* and p_2^* then satisfy certain conditions (in particular, if $p_1^* > 1/2$) then the $\overline{\Pi}$-maximizing MNQ contract induces a declining effort profile (see Lemma E(b)).

In particular, the uniform, exponential, and power-law families of distributions all satisfy the technical conditions required by Lemma E; thus, for all three families of distributions, the $\overline{\Pi}$-maximizing element in the space of MNQ contract is tenure-track, and induces a declining profile of effort (see Lemmas F, G, and H). In other words, the conclusions of Theorem 1 hold for the space of MNQ contracts. Then Proposition 4(b) implies that the conclusions of Theorem 1 hold for the space of *all* contracts. Finally, Proposition 3 implies that the conclusions of Theorem 1 hold for the restricted space of raid-proof contracts; this establishes Theorem 1.

V Concluding Remarks

This paper provides an explanation of the use of tenure-track contracts in academia that arises out of the unique nature of academic productivity and optimizing behavior on the part of the university. The theory, briefly, is that, because a university's mission involves encouraging its faculty to engage in research that is important but yields no saleable results, a professor's marginal revenue product does not depend simply on her current research production. Rather, because her research accomplishments act as a signal of knowledge that serves to attract tuition-paying students, a professor's contribution to the university's revenues, at any point in her career, will depend on the strength of her cumulative research record. The university then profits by dismissing a professor who fails to establish a strong research record initially, but by retaining a professor who establishes a strong record regardless of her research output thereafter.

The theory further provides a simple explanation for the observation that academic research production declines, on average, with age. Because the university's opportunities to realize tuition revenues from a professor's spot research accomplishments diminish as she approaches the end of her career, the optimal level of induced research effort, and therefore the expected level of research output, diminishes with age.

Tenure, of course, does not amount to absolute job security. While tenured professors are not dismissed for poor research productivity, Lovain (1983/84), Hendrickson (1988) and Morris (1992) note that they *are* dismissed for failing to perform their teaching duties. Our theory provides a simple explanation: the past research accomplishments of a tenured professor can be translated into the tuition revenues necessary to make her profitable only if she continues to teach.

The theory also serves to correct some common misperceptions. In particular, our analysis shows that declining research production over the life cycle does not reflect some disincentive effect of tenure, and tenure itself is not a measure of security that a university concedes in lieu of compensation or with reluctance to a powerful faculty union.

The most important implication of our theory is that the tolerance for research failure that characterizes tenure *is* consistent with a university's interest in advancing knowledge through research production. Although it might seem that a university could produce more research simply by replacing *any* unproductive scholar, or by providing older professors with greater research incentives, our analysis suggests that, by deviating from its profit-maximizing rule, either the university's long-term viability would be undermined or greater levels of ongoing research subsidies would be required.

Similarly, if, under the pressure of system-wide funding constraints, universities as a group were to abolish tenure or adopt post-tenure reviews, our analysis suggests that an efficiency loss would result as the full value of past research accomplishments would go unrealized.

Appendix: Proofs

Proof of Lemma 2. Let $\overline{\Pi}$ be the expected profit per period under $\mathbf{C}$, as defined in eqn.(19). Let $\widetilde{\Pi}$ be the expected profit per period under $\widetilde{\mathbf{C}}$. Then clearly

$$\overline{\Pi} \ = \ \overline{R} - \overline{C} \quad \text{and} \quad \widetilde{\Pi} \ = \ \widetilde{R} - \widetilde{C}, \tag{2.1}$$

where $\overline{R}$ and $\widetilde{R}$ represent the university's expected *revenue* per period under the two contracts, while $\overline{C}$ and $\widetilde{C}$ represent the university's expected *costs* per period.

(Eq1) implies that the professor will exhibit the same probability distribution of research outputs; in particular she will have the same expected values $R_1^* := \overline{R}(e_1^*, s_1)$ and $R_2^* := \overline{R}(e_2^*, s_2)$. Then (Eq2) implies she will have the same retention probabilities (p_1, p_2)

in both contracts. Thus equation (17) says both contracts have the same stationary probability distribution (π_1, π_2, π_3) over the three periods. Thus, assuming $k_1 = k_2 = k$, both contracts generate the same expected revenue per period, namely

$$\widetilde{R} \quad = \quad \pi_1 \cdot 0 + \pi_2 \cdot k\, R_1^* + \pi_3 \cdot k\,(R_1^* + R_2^*) \quad = \quad \overline{R}. \tag{2.2}$$

Let $\overline{S}_1, \overline{S}_2, \overline{S}_3$ denote the professor's expected salaries in the three periods, under **C**. Then $\overline{C}$ is simply the professor's expected salary per period, namely:

$$\overline{C} \quad = \quad \pi_1\overline{S}_1 + \pi_2\overline{S}_2 + \pi_3\overline{S}_3 \quad \underset{(17)}{=\!=} \quad \frac{\overline{S}_1 + p_1(\overline{S}_2 + p_2\overline{S}_3)}{1 + p_1 + p_1p_2} \quad = \quad \frac{\overline{S}}{1 + p_1 + p_1p_2},$$

where $\overline{S} := \overline{S}_1 + p_1(\overline{S}_2 + p_2\overline{S}_3)$ is the professor's expected lifetime salary in **C**. Likewise, $\widetilde{C} := \widetilde{S}/(1 + p_1 + p_1p_2)$, where $\widetilde{S}$ is the professor's lifetime salary in $\widetilde{\mathbf{C}}$. The professor's expected lifetime net benefit under the two contracts can be expressed by

$$\mathrm{NB}_{1,2,3} \quad = \quad \overline{S} - (e_1^*)^2 - p_1 \cdot (e_2^*)^2 \quad \text{and} \quad \widetilde{\mathrm{NB}}_{1,2,3} \quad = \quad \widetilde{S} - (e_1^*)^2 - p_1 \cdot (e_2^*)^2.$$

But (Eq3) says $\widetilde{\mathrm{NB}}_{1,2,3} = \mathrm{NB}_{1,2,3}$; hence $\widetilde{S} = \overline{S}$; hence $\widetilde{C} = \overline{C}$. Combining this with equations (2.1) and (2.2), we get $\widetilde{\Pi} = \overline{\Pi}$. □

Proof of Proposition 3. Let (e_1^*, e_2^*) be the utility-maximizing effort profile for **C**. Let $r_1^* := \overline{r}(e_1^*)$ and $r_2^* := \overline{r}(e_2^*)$. If $\widetilde{\mathbf{C}}$ is equivalent to **C**, then (e_1^*, e_2^*) will also be the utility-maximizing effort profile for $\widetilde{\mathbf{C}}$ (we will ensure this later). In that case, the expected profit of $\widetilde{\mathbf{C}}$ *before* each period will be given by:

$$\begin{aligned} \overline{\Pi}_1 \quad &\underset{(13)}{=\!=} \quad -w_1 - b_1r_1^*; \\ \overline{\Pi}_2(r_1) \quad &\underset{(14)}{=\!=} \quad -w_2 - b_2r_2^* + (k - b_{21})r_1; \\ \text{and} \quad \overline{\Pi}_3(r_1, r_2) \quad &\underset{(15)}{=\!=} \quad -w_3 + (k - b_{31})r_1 + (k - b_{32})r_2. \end{aligned}$$

To make $\widetilde{\mathbf{C}}$ raid-proof, it suffices to ensure that $\overline{\Pi}_3(r_1, r_2) = \overline{\Pi}_2(r_1) = \overline{\Pi}_1$ for all $r_1, r_2 \geq 0$. To do this, we must set

$$b_{21} \quad := \quad b_{31} \quad := \quad b_{32} \quad := \quad k; \tag{3.1}$$

$$w_3 \quad := \quad w_1 + b_1r_1^*; \qquad \text{and} \tag{3.2}$$

$$w_2 \quad := \quad w_1 + b_1r_1^* - b_2r_2^*. \tag{3.3}$$

The net benefit of contract $\widetilde{\mathbf{C}}$ for the professor during period 3 is then

$$\widetilde{\mathrm{NB}}_3(r_1, r_2) \quad = \quad w_3 + kr_1 + kr_2, \qquad \text{by (5) and (3.1).} \tag{3.4}$$

At the beginning of period 2, the value of r_1 is known, and the expected future value of $\widetilde{\mathrm{NB}}_3$, as a function of e_2, is given:

$$\widetilde{\mathrm{NB}}_3(r_1, e_2) \quad \underset{(3.4)}{=\!=} \quad w_3 + kr_1 + k\overline{r}(e_2). \tag{3.5}$$

Let $\widetilde{\mathrm{NB}}_{2,3}$ be the net benefit of $\widetilde{\mathbf{C}}$ at the start of period 2 (including the anticipated future benefit of period 3). By hypothesis, $\mathbf{C}$ is tenure-track (i.e. $p_2 = 1$); hence, to be equivalent, $\widetilde{\mathbf{C}}$ must also be tenure-track. In this case, the expected value of $\widetilde{\mathrm{NB}}_{2,3}$ at the beginning of period 2, as a function of e_2, is given:

$$\begin{aligned}
\widetilde{\mathrm{NB}}_{2,3}(r_1, e_2) \quad & \underset{(7)}{=} \quad \widetilde{\mathrm{NB}}_2(r_1, e_2) + \widetilde{\mathrm{NB}}_3(r_1, e_2) \\
& \underset{(8,3.1)}{=} \quad w_2 + b_2\overline{r}(e_2) + kr_1 + \widetilde{\mathrm{NB}}_3(r_1, e_2) - e_2^2 \\
& \underset{(3.5)}{=} \quad (w_2 + w_3) + 2k\,r_1 + (k + b_2)\overline{r}(e_2) - e_2^2 \\
& \underset{(3.2,3.3)}{=} \quad 2w_1 + 2b_1 r_1^* - b_2 r_2^* + 2k\,r_1 + (k + b_2)\overline{r}(e_2) - e_2^2. \qquad (3.6)
\end{aligned}$$

Let s_1 be the period 1 standard of $\mathbf{C}$ (and hence, of $\widetilde{\mathbf{C}}$). If the professor exerts effort e_1 during period 1, and is retained during period 2, then the conditionally expected value of r_1, given this information, is $\overline{R}(e_1) := \overline{R}(e, s_1)$ [see eqn.(4)]. Thus, the expected future value of $\widetilde{\mathrm{NB}}_{2,3}$ at the beginning of period 1, as a function of e_1 and e_2, is given:

$$\widetilde{\mathrm{NB}}_{2,3}(e_1, e_2) \quad \underset{(3.6)}{=} \quad 2w_1 + 2b_1 r_1^* - b_2 r_2^* + 2k\,\overline{R}(e_1) + (k + b_2)\overline{r}(e_2) - e_2^2. \qquad (3.7)$$

Let $\widetilde{\mathrm{NB}}_{1,2,3}$ be the lifetime net benefit of $\widetilde{\mathbf{C}}$ at the start of period 1 (including the anticipated potential future benefits in periods 2 and 3). For any $e \geq 0$, let $P(e) := p(e, s_1)$ [see eqn.(4)]. Thus, the expected value of $\widetilde{\mathrm{NB}}_{1,2,3}$, as a function of e_1 and e_2, is

$$\begin{aligned}
\widetilde{\mathrm{NB}}_{1,2,3}(e_1, e_2) \quad & \underset{(10,11)}{=} \quad w_1 + b_1\overline{r}(e_1) + P(e_1) \cdot \widetilde{\mathrm{NB}}_{2,3}(e_1, e_2) - e_1^2 \\
\underset{(3.7)}{=} \quad & w_1 + b_1\overline{r}(e_1) + P(e_1)\Big(2w_1 + 2b_1 r_1^* - b_2 r_2^* + 2k\,\overline{R}(e_1) + (k + b_2)\overline{r}(e_2) - e_2^2\Big) - e_1^2 \\
= \quad & \Big(1 + 2P(e_1)\Big)\, w_1 + b_1\overline{r}(e_1) - e_1^2 \\
& \qquad + \; P(e_1)\Big(2b_1 r_1^* - b_2 r_2^* + 2k\,\overline{R}(e_1) + (k + b_2)\overline{r}(e_2) - e_2^2\Big). \qquad (3.8)
\end{aligned}$$

Let $p_1 := P(e_1^*, s_1)$ and let $\overline{R}_1^* := \overline{R}(e_1^*)$. If the professor exerted effort profile (e_1^*, e_2^*), then the expected lifetime net benefit of $\widetilde{\mathbf{C}}$ would be

$$\begin{aligned}
& \widetilde{\mathrm{NB}}_{1,2,3}(e_1^*, e_2^*) \\
& \quad \underset{(3.8)}{=} \quad \Big(1 + 2P(e_1^*)\Big)\, w_1 + b_1\overline{r}(e_1^*) - (e_1^*)^2 \\
& \qquad\qquad + \; P(e_1^*)\Big(2b_1 r_1^* - b_2 r_2^* + 2k\,\overline{R}(e_1^*) + (k + b_2)\overline{r}(e_2^*) - (e_2^*)^2\Big) \\
& \quad = \quad (1 + 2p_1)w_1 + b_1 r_1^* - (e_1^*)^2 + p_1\Big(2b_1 r_1^* + 2k\,\overline{R}_1^* + kr_2^* - (e_2^*)^2\Big). \qquad (3.9)
\end{aligned}$$

The expected lifetime net benefit offered by contract $\mathbf{C}$ is $\mathrm{NB}_{1,2,3} = 0$, because $\mathbf{C}$ is admissible by hypothesis. We must also make $\widetilde{\mathrm{NB}}_{1,2,3} = 0$. For any values of b_1 and b_2, we can achieve this by setting

$$w_1 \quad = \quad w_1(b_1) \quad := \quad \frac{-b_1 r_1^* - p_1\Big(2b_1 r_1^* + 2k\,\overline{R}_1^* + kr_2^* - (e_2^*)^2\Big) + (e_1^*)^2}{1 + 2p_1}. \qquad (3.10)$$

At this point, $\widetilde{\mathbf{C}}$ has only two free parameters: b_1 and b_2. Substituting eqn.(3.10) into (3.7) and (3.8), we define, for all $b_1, b_2 \in \mathbb{R}$, the functions

$$\begin{aligned}
&\widetilde{\mathrm{NB}}_{2,3}(b_1, b_2; e_1, e_2) \\
&\quad := \; 2w_1(b_1) + 2b_1 r_1^* - b_2 r_2^* + 2k\,\overline{R}(e_1) + (k + b_2)\overline{r}(e_2) - e_2^2, \qquad \text{and} \qquad (3.11) \\
&\widetilde{\mathrm{NB}}_{1,2,3}(b_1, b_2; e_1, e_2) \; := \; \Big(1 + 2P_1(e_1)\Big)\, w_1(b_1) + b_1\overline{r}(e_1) - e_1^2 \\
&\qquad + P(e_1)\left(2b_1 r_1^* - b_2 r_2^* + 2k\,\overline{R}(e_1) + (k + b_2)\overline{r}(e_2) - e_2^2\right). \qquad (3.12)
\end{aligned}$$

Now we must choose b_1, b_2 so that the effort profile (e_1^*, e_2^*) is still optimal for the professor under contract $\widetilde{\mathbf{C}}$. That is, we must ensure that

$$\partial_{e_2}\, \widetilde{\mathrm{NB}}_{2,3}(b_1, b_2; e_1^*, e_2^*) \;=\; 0 \quad \text{and} \quad \partial_{e_1}\, \widetilde{\mathrm{NB}}_{1,2,3}(b_1, b_2; e_1^*, e_2^*) \;=\; 0; \tag{3.13}$$

Differentiating eqn.(3.11) we get $\partial_{e_2}\, \widetilde{\mathrm{NB}}_{2,3}(b_1, b_2; e_1^*, e_2^*) = (k + b_2)\overline{r}'(e_2^*) - 2e_2^*$. Thus, we have $\partial_{e_2}\, \widetilde{\mathrm{NB}}_{2,3}(b_1, b_2; e_1^*, e_2^*) = 0$ if and only if

$$b_2 = \frac{2e_2^*}{\overline{r}'(e_2^*)} - k. \tag{3.14}$$

Differentating eqn.(3.12), we get a (complicated) expression for $\partial_{e_1}\, \widetilde{\mathrm{NB}}_{2,3}(b_1, b_2; e_1^*, e_2^*)$. Solving for b_1 to satisfy eqn.(3.13), we get

$$b_1 \;=\; \frac{B}{\overline{r}'(e_1^*)\,(2\,p_1 + 1)}, \tag{3.15}$$

where $B \;:=\; 4\,P'(e_1^*)p_1\,k\overline{R}_1^* + 2\,P'(e_1^*)p_1\,kr_2^* - 2\,P'(e_1^*)p_1\,(e_2^*)^2 - 2\,P'(e_1^*)(e_1^*)^2 + 2\,e_1^* + 4\,e_1^*p_1 + P'(e_1^*)\,b_2\,r_2^* + 2\,P'(e_1^*)\,b_2\,r_2^*\,p_1 - 2\,P'(e_1^*)k\overline{R}(e_1^*) - 4\,P'(e_1^*)k\overline{R}(e_1^*)p_1 - P'(e_1^*)r(e_2^*)k - 2\,P'(e_1^*)r(e_2^*)kp_1 - P'(e_1^*)r(e_2^*)\,b_2 - 2\,P'(e_1^*)r(e_2^*)\,b_2\,p_1 + P'(e_1^*)(e_2^*)^2 + 2\,P'(e_1^*)(e_2^*)^2p_1 - 2\,P(e_1^*)k\overline{R}'(e_1^*) - 4\,P(e_1^*)k\overline{R}'(e_1^*)p_1$.

Proof of contract equivalence. The expressions (3.14) and (3.15) are well-defined because $\overline{r}'(e_2^*) \neq 0$ and $\overline{r}'(e_1^*) \neq 0$ by hypothesis. If we define b_1 and b_2 as in (3.14) and (3.15), then the equations (3.13) hold, so the professor's optimal effort profile is (e_1^*, e_2^*), as desired. Thus, condition (Eq1) is satisfied. If we then substitute the value of $w_1(b_1)$ from eqn.(3.10) into expression (3.9), we will get $\widetilde{\mathrm{NB}}_{1,2,3} = 0 = \mathrm{NB}_{1,2,3}$; thus, condition (Eq3) is satisfied. Condition (Eq2) is satisfied automatically because we have assumed that both $\mathbf{C}$ and $\widetilde{\mathbf{C}}$ have the same value for s_1, and set $s_2 = 0$.

Proof that $\widetilde{\mathbf{C}}$ is admissible. $\widetilde{\mathbf{C}}$ satisfies (12) because $\mathbf{C}$ does, by condition (Eq3). Now, $\mathbf{C}$ also satisfies the 'no quitting' constraints (6) and (9), so $\mathrm{NB}_{2,3} \geq 0$ and $\mathrm{NB}_3 \geq 0$; thus, it suffices to show that $\widetilde{\mathrm{NB}}_{2,3} \geq \mathrm{NB}_{2,3}$ and $\widetilde{\mathrm{NB}}_3 \geq \mathrm{NB}_3$. To do this, first note that (5) implies

$$\widetilde{\mathrm{NB}}_3 - \mathrm{NB}_3 \quad = \quad \widetilde{S}_3 - \overline{S}_3. \tag{3.16}$$

Also, $\widetilde{\mathbf{C}}$ and $\mathbf{C}$ induce the same effort profile (e_1^*, e_2^*); thus, the professor experiences the same disutility of effort $(e_2^*)^2$ in period 2 of both contracts; thus, equation (8) implies that $\widetilde{\mathrm{NB}}_2 - \mathrm{NB}_2 = \widetilde{S}_2 - \overline{S}_2$. Furthermore, $p_2 = 1$ in both contracts; thus, equation (7) implies that

$$\widetilde{\mathrm{NB}}_{2,3} - \mathrm{NB}_{2,3} \;=\; (\widetilde{\mathrm{NB}}_2 - \mathrm{NB}_2) + (\widetilde{\mathrm{NB}}_3 - \mathrm{NB}_3) \;\underset{(3.16)}{=\!=}\; (\widetilde{S}_2 - \overline{S}_2) + (\widetilde{S}_3 - \overline{S}_3). \quad (3.17)$$

Lemma 2 says $\overline{\Pi} = \widetilde{\Pi}$. But $\widetilde{\mathbf{C}}$ is raid-proof, while $\mathbf{C}$ was not. This means we must have $\widetilde{\Pi}_1 \geq \overline{\Pi}_1$, while $\widetilde{\Pi}_2 \leq \overline{\Pi}_2$ and $\widetilde{\Pi}_3 \leq \overline{\Pi}_3$. Since both contracts yield the same expected revenue (2.2) in each period, this can only mean that $\widetilde{S}_2 \geq \overline{S}_2$ and $\widetilde{S}_3 \geq \overline{S}_3$. Substituting this into equations (3.16) and (3.17) yields $\widetilde{\mathrm{NB}}_3 - \mathrm{NB}_3 \geq 0$ and $\widetilde{\mathrm{NB}}_{2,3} - \mathrm{NB}_{2,3} \geq 0$; hence $\widetilde{\mathbf{C}}$ satisfies (6) and (9). □

To prove Proposition 4, we need the following lemma.

Lemma A *Suppose contract* $\mathbf{C}$ *satisfies minimal recruitment condition* (12) *and constraint* (MNQ), *and suppose* $\{\rho_e\}_{e\in\mathbb{R}_{\not -}}$ *satisfies* (B). *Define* $\epsilon := \beta^{-1} : (0,\infty)\longrightarrow(0,\infty)$.

(a) *The professor's optimal effort profile is given by* $e_1^* = \epsilon(b_1)$ *and* $e_2^* = \epsilon(b_2)$.

(b) *Let* $\omega(b) := \epsilon(b)^2 - b\,\overline{r}[\epsilon(b)]$. *Then* $\mathbf{C}$ *must have* $b_{12} = b_{13} = b_{23} = w_3 = 0$, $w_2 = \omega(b_2)$, *and* $w_1 = \omega(b_1)$.

Proof: Hypothesis (B) implies β is invertible. Examining eqn.(5) reveals that, to make $\mathrm{NB}_3 = 0$ for all $r_1, r_2 \geq 0$, we must set $b_{13} := b_{23} := w_3 := 0$. We then have

$$\mathrm{NB}_{2,3}(r_1, e_2) \;\underset{(7)}{=}\; \mathrm{NB}_2(r_1, e_2) \;\underset{(8)}{=}\; w_2 + b_2\overline{r}(e_2) + b_{21}r_1 - e_2^2.$$

Thus, the optimal effort e_2^* is the solution to the equation $b_2\overline{r}'(e_2) = 2e_2$. It is easy to check that $e_2^* := \epsilon(b_2)$ is the unique solution to this equation. To ensure that $\mathrm{NB}_2 = 0$ for all $r_1 \geq 0$, we must then set $b_{21} := 0$ and set $w_2 = \omega(b_2)$. We then have

$$\mathrm{NB}_{1,2,3}(e_1, e_2^*) \;\underset{(10)}{=\!=}\; \mathrm{NB}_1(e_1) \;\underset{(11)}{=\!=}\; w_1 + b_1\overline{r}(e_1) - e_1^2.$$

Thus, e_1^* is the solution to the equation $b_1\overline{r}'(e_1) = 2e_1$; again, the unique solution is $e_1^* := \epsilon(b_1)$. If we finally set $w_1 = \omega(b_1)$, then we satisfy (12). □

Proof of Proposition 4. (a) Suppose $\mathbf{C}$ has optimal effort profile (e_1^*, e_2^*) and standards (s_1, s_2). Let $\widetilde{\mathbf{C}}$ have the same standards (s_1, s_2) (so that (Eq2) is satisfied), and set $b_1 := \beta(e_1^*)$, $b_2 := \beta(e_2^*)$, $b_{12} = b_{13} = b_{23} = w_3 = 0$, $w_2 = \omega(b_2)$, and $w_1 = \omega(b_1)$. Lemma A says that $\widetilde{\mathbf{C}}$ is a MNQ contract which also has optimal effort profile (e_1^*, e_2^*). Thus, (Eq1) is satisfied. Lemma A also says that $\widetilde{\mathbf{C}}$ satisfies (12); thus (Eq3) is satisfied.

(b) If $\mathbf{C}$ is the globally $\overline{\Pi}$-maximizing contract, then part (a) yields an MNQ contract $\widetilde{\mathbf{C}}$ which is equivalent to $\mathbf{C}$, hence yields the same value of $\overline{\Pi}$ (by Lemma 2), hence is also $\overline{\Pi}$-maximixing. If $\mathbf{C}$ satisfies (12), then so does $\widetilde{\mathbf{C}}$, by (Eq3). Finally, any MNQ contract automatically satisfies (6) and (9); thus, $\widetilde{\mathbf{C}}$ is admissible. □

For any $e \geq 0$, define $P_e(s) := P(e,s)$. Then $P_e : [0,\infty)\longrightarrow(0,1]$ is a strictly decreasing bijection; hence invertible. Define $\varsigma(e,p) := P_e^{-1}(p)$. It is easy to prove the next result.

Lemma B *For any $e_1, e_2 \geq 0$ and $p_1, p_2 \in [0,1]$, we can achieve the effort profile (e_1, e_2) and retention probabilities (p_1, p_2) with the MNQ contract $(b_1, b_2; s_1, s_2)$ defined by $b_k = \beta(e_k)$ and $s_k = \varsigma(e_k, p_k)$.* □

If $b_{12} = b_{13} = b_{23} = w_3 = 0$, with $w_2 = \omega(b_2)$, and $w_1 = \omega(b_1)$ as specified in Lemma A, and b_1, b_2, s_1 and s_2 are as specified in Lemma B, then equations (18) become:

$$\begin{array}{rcl} \overline{\Pi}_1(e_1,e_2;p_1,p_2) & = & -e_1^2; \\ \overline{\Pi}_2(e_1,e_2;p_1,p_2) & = & k\,\widetilde{R}(e_1,p_1) - e_2^2; \\ \text{and}\quad \overline{\Pi}_3(e_1,e_2;p_1,p_2) & = & k\,\widetilde{R}(e_1,p_1) + k\,\widetilde{R}(e_2,p_2), \end{array} \tag{B.1}$$

where $\widetilde{R}(e,p) := \overline{R}[e,\varsigma(e,p)]$. Substituting (B.1) and (17) into (19), the expected profit for the University is given by

$$\overline{\Pi}(e_1,e_2;p_1,p_2) \;=\; \frac{-e_1^2 + p_1\left(-e_2^2 + k\,\widetilde{R}(e_1,p_1)\right) + p_1p_2k\,\left(\widetilde{R}(e_1,p_1) + \widetilde{R}(e_2,,p_2)\right)}{1+p_1+p_1p_2}. \tag{B.2}$$

Lemma C *Assume hypothesis* (B). *For any $p \in [0,1]$ and $e > 0$, define $\gamma_p(e) := e/\partial_1\,\widetilde{R}(e,p)$. Suppose that, for all $p \in [0,1]$, the function $\gamma_p : \mathbb{R}_{\not -}\longrightarrow\mathbb{R}_{\not -}$ is bijective.*

(a) *For any fixed (p_1,p_2), the values of (e_1,e_2) which maximize the value of $\overline{\Pi}(e_1,e_2;p_1,p_2)$ are given by*

$$e_1^*(p_1,p_2) \;:=\; \gamma_{p_1}^{-1}\left(\frac{k\,p_1(1+p_2)}{2}\right) \qquad \textit{and} \qquad e_2^*(p_2) \;:=\; \gamma_{p_2}^{-1}\left(\frac{k\,p_2}{2}\right). \tag{C.1}$$

(b) *In particular, suppose $\widetilde{R}(e,p) = e\,L(p)$ for some function $L : [0,1]\longrightarrow\mathbb{R}_+$. Then $e_1^*(p_1,p_2) \;=\; L(p_1)\,k\,p_1(1+p_2)/2$ and $e_2^*(p_2) \;=\; L(p_2)\,k\,p_2/2$.*

Proof: (a) Differentiating (B.2) we get

$$\begin{array}{rrcl} & \partial_{e_1}\,\overline{\Pi}(e_1,e_2;p_1,p_2) & = & \dfrac{-2e_1 + k\,(p_1+p_1p_2)\,\partial_1\,\widetilde{R}(e_1,p_1)}{1+p_1+p_1p_2} \\ \text{and} & \partial_{e_2}\,\overline{\Pi}(e_1,e_2;p_1,p_2) & = & \dfrac{-2p_1e_2 + k\,p_1p_2\,\partial_1\,\widetilde{R}(e_2,p_2)}{1+p_1+p_1p_2}. \end{array}$$

To make the numerators of these expressions zero, we need

$$\frac{e_1}{\partial_1\,\widetilde{R}(e_1,p_1)} \;=\; \frac{k\,(p_1+p_1p_2)}{2} \quad\text{and}\quad \frac{e_2}{\partial_1\,\widetilde{R}(e_2,p_2)} \;=\; \frac{k\,p_2}{2},$$

which is achieved by eqn.(C.1).

(b) If $\widetilde{R}(e,p) = e\,L(p)$, then $\partial_1\,\widetilde{R}(e,p) = L(p)$, so $\gamma_p(e) = e/L(p)$, so $\gamma_p^{-1}(x) = L(p)\,x$. Now apply part (a). □

If the hypotheses of Lemma C are satisfied, then the $\overline{\Pi}$-maximization problem is reduced to finding the $(p_1, p_2) \in [0, 1]^2$ which maximize the function

$$\hat{\Pi}(p_1, p_2) \quad := \quad \overline{\Pi}[e_1^*(p_1, p_2), e_2^*(p_1, p_2); p_1, p_2]. \tag{C.2}$$

The family of distributions $\{\rho_e\}_{e \in \mathbb{R}_{\neq}}$ is *tenable* if it satisfies two conditions:

(T1) $\overline{R}(e, s) = c_1 e + c_2 s$ for some constants $c_1, c_2 \in \mathbb{R}_+$.

(T2) $\varsigma(e, p) = e\, S(p)$ for some function $S : [0, 1] \longrightarrow \mathbb{R}_+$, with $S(1) = 0$.

'Tenability' is a technical condition with no obvious economic interpretation. However, we will later see that all three distribution families in Theorem 1 are tenable.

Lemma D *Suppose* $\{\rho_e\}_{e \in \mathbb{R}_{\neq}}$ *is tenable. Then hypothesis* (B) *holds. Define* $L(p) := c_1 + c_2\, S(p)$*. Then* $\widetilde{R}(e, p) = e\, L(p)$*, so Lemma* C(b) *applies. Furthermore,*

$$\hat{\Pi}(p_1, p_2) \quad = \quad \frac{k^2\, p_1}{4} \left(\frac{p_1 L(p_1)^2 (1 + p_2)^2 + L(p_2)^2 p_2^2}{1 + p_1 + p_1 p_2} \right). \tag{D.1}$$

Thus,

$$\partial_2\, \hat{\Pi}(p_1, p_2) \quad = \quad \frac{k^2\, p_1\, \Xi(p_1, p_2)}{4(1 + p_1 + p_1 p_2)^2}, \qquad \textit{where} \tag{D.2}$$

$$\begin{aligned} \Xi(p_1, p_2) \quad := \quad & 2(1 + p_1 + p_1 p_2) \left(p_1\, L(p_1)^2 (1 + p_2) + L(p_2) L'(p_2) p_2^2 + L(p_2)^2 p_2 \right) \quad \text{(D.3)} \\ & - p_1 \left(p_1 L(p_1)^2 (1 + p_2)^2 + L(p_2)^2 p_2^2 \right). \end{aligned}$$

Proof: For any $e \geq 0$, we have $\overline{r}(e) = \overline{R}(e, 0) \underset{\text{(T1)}}{=\!=} c_1 e$; thus, $\overline{r}'(e) = c_1 > 0$ is constant, so $\beta(e) := 2e/\overline{r}'(e) = 2e/c_1$ satisfies condition (B). Now,

$$\begin{aligned} \widetilde{R}(e, p) \quad &= \quad \overline{R}[e, \varsigma(e, p)] \underset{\text{(T1)}}{=\!=} c_1 e + c_2 \varsigma(e, p) \underset{\text{(T2)}}{=\!=} c_1 e + c_2 e\, S(p) \\ &= \quad e(c_1 + c_2 S(p)) = e\, L(p). \end{aligned} \tag{D.4}$$

Equation (D.4) means that Lemma C(b) is applicable, so the functions $e_1^*(p_1, p_2)$ and $e_2^*(p_2)$ are well-defined. We define

$$\widehat{R}_1(p_1, p_2) \quad := \quad \widetilde{R}[e_1^*(p_1, p_2), p_1] \underset{\text{(D.4)}}{=\!=} e_1^*(p_1, p_2)\, L(p_1), \tag{D.5}$$

$$\text{and} \quad \widehat{R}_2(p_2) \quad := \quad \widetilde{R}[e_2^*(p_2), p_2] \underset{\text{(D.4)}}{=\!=} e_2^*(p_2)\, L(p_2). \tag{D.6}$$

Substitute (B.2), (D.5) and (D.6) into (C.2) to obtain

$$\begin{aligned} \hat{\Pi}(p_1, p_2) \quad &= \quad \frac{-e_1^*(p_1, p_2)^2 + p_1 \left(-e_2^*(p_2)^2 + k\, \widehat{R}_1(p_1, p_2) \right) + k\, p_1 p_2 \left(\widehat{R}_1(p_1, p_2) + \widehat{R}_2(p_2) \right)}{1 + p_1 + p_1 p_2} \\ &= \quad \frac{k p_1 (1 + p_2) \widehat{R}_1(p_1, p_2) - e_1^*(p_1, p_2)^2}{1 + p_1 + p_1 p_2} \; + \; p_1 \left(\frac{k p_2 \widehat{R}_2(p_2) - e_2^*(p_2)^2}{1 + p_1 + p_1 p_2} \right). \end{aligned} \tag{D.7}$$

Now,

$$\begin{aligned}
kp_1(1+p_2)\widehat{R}_1(p_1,p_2) - e_1^*(p_1,p_2)^2 &\underset{(D.5)}{=\!=} kp_1(1+p_2)\,e_1^*(p_1,p_2)\,L(p_1) - e_1^*(p_1,p_2)^2 \\
&\underset{(*)}{=} k^2p_1^2(1+p_2)^2L(p_1)^2/2 - k^2p_1^2(1+p_2)^2L(p_1)^2/4 \\
&= k^2p_1^2(1+p_2)^2L(p_1)^2/4. \qquad \text{(D.8)} \\
\text{and} \quad kp_2\widehat{R}_2(p_2) - e_2^*(p_2)^2 &\underset{(D.6)}{=\!=} kp_2\,e_2^*(p_2)\,L(p_2) - e_2^*(p_2)^2 \\
&\underset{(*)}{=} L(p_2)^2\,k^2\,p_2^2/2 - L(p_2)^2\,k^2\,p_2^2/4 \\
&= L(p_2)^2\,k^2\,p_2^2/4. \qquad \text{(D.9)}
\end{aligned}$$

where $(*)$ is Lemma C(b). Substituting (D.8) and (D.9) into (D.7) we get

$$\hat{\Pi}(p_1,p_2) \quad = \quad \frac{k^2p_1^2(1+p_2)^2L(p_1)^2 \ + \ p_1L(p_2)^2\,k^2\,p_2^2}{4(1+p_1+p_1p_2)},$$

which we factor to obtain (D.1). Differentiating (D.1) yields (D.2). □

Lemma E *Suppose* $\{\rho_e\}_{e\in\mathbb{R}_{\neq}}$ *is tenable, and let* Ξ *be as in equation* (D.3)*.*

- **(a)** *Suppose* $\Xi(p_1,p_2) \geq 0$ *for all* $(p_1,p_2) \in [0,1]^2$*. Then the* $\overline{\Pi}$*-maximizing MNQ contract is tenure-track* (i.e. $p_2^* = 1$)*.*
- **(b)** *In this case* $e_1^* = k(c_1 + c_2S(p_1^*))p_1^*$ *and* $e_2^* = kc_1/2$*. Thus, if* $S(p_1) > c_1(1-2p_1)/2c_2p_1$ *then the* $\overline{\Pi}$*-maximizing MNQ contract induces a declining effort profile* (i.e. $e_1^* > e_2^*$)*. In particular, if* $p_1^* > 1/2$*, then* $e_1^* > e_2^*$*.*

Proof: Part (a) follows immediately from eqn.(D.2). Part (b) follows by substituting $p_2^* = 1$ into Lemma C(b); note that $L(1) = c_1 + c_2S(1) = c_1$, because $S(1) = 0$. □

We are now in a position to prove the equivalent of Theorem 1 in the restricted setting onf MNQ contracts. This is the content of the next three lemmas.

Lemma F *Suppose* $\{\rho_e\}_{e\in\mathbb{R}_{\neq}}$ *is the family of uniform distributions from* Theorem 1(a)*. Then the* $\overline{\Pi}$*-maximizing MNQ contract is tenure-track, with a declining effort profile.*

Proof: For all $0 \leq s \leq e$ we have $P(e,s) = (e-s)/e$; hence $\varsigma(e,p) = e(1-p)$. Also, $\overline{R}(e,s) = (e+s)/2$. Thus, setting $c_1 = c_2 = \frac{1}{2}$ and $S(p) = 1-p$, we see that $\{\rho_e\}$ is tenable, so we can apply Corollary E. We have $L(p) = (2-p)/2$ in Lemma D. Substitute this expression for $L(p)$ into eqn.(D.3) to get $\Xi(p_1,p_2) = f(p_1,p_2)/4$, where

$$\begin{aligned}
f(p_1,p_2) \;:=\; & 16\,p_1p_2 + 8\,p_1 + 8\,p_2 - 12\,p_2^2 - 6\,p_1^3p_2 - 4\,p_1^2 - 2\,p_1^3 + 4\,p_2^3 - 8\,p_1p_2^2 \\
& + 4\,p_1^2p_2^2 - 4\,p_1^3p_2^2 + 2\,p_1^4p_2 + p_1^4p_2^2 + p_1^4 - 4\,p_1p_2^3 + 3\,p_1p_2^4.
\end{aligned}$$

Claim 1: $f(p_1,p_2) \geq 0$ *for all* $(p_1,p_2) \in [0,1]^2$*.*

Proof: Let

$$\begin{aligned} g(p_1,p_2) &:= 16\,p_1p_2 + 8\,p_1 + 8\,p_2 - 12\,p_2^2 - 6\,\underline{p_1^1}p_2 - 4\,\underline{p_1^1} - 2\,\underline{p_1^1} + 4\,p_2^3 - 8\,p_1p_2^2 \\ &\quad + 4\,p_1^2p_2^2 - 4\,\underline{p_1^2}p_2^2 + 2\,p_1^4p_2 + p_1^4p_2^2 + p_1^4 - 4\,p_1\underline{p_2^2} + 3\,p_1p_2^4 \\ &= 10\,p_1p_2 + 2\,p_1 + 8\,p_2 - 12\,p_2^2 + 4\,p_2^3 - 12\,p_1p_2^2 + 2\,p_1^4p_2 + p_1^4p_2^2 + p_1^4 + 3\,p_1p_2^4. \end{aligned}$$

Claim 1.1: *$g(p_1,p_2) \le f(p_1,p_2)$, for all $(p_1,p_2) \in [0,1]^2$.*

Proof: Suppose $0 < n < m$. If $0 \le x \le 1$ then $x^n \ge x^m$; hence $-x^n \le -x^m$. We obtained $g(p_1,p_2)$ by taking the expression for $f(p_1,p_2)$ and decreasing the exponents on the underlined negative terms. Each of these terms is made smaller by this change (by previous paragraph); thus, $g(p_1,p_2) \le f(p_1,p_2)$. $\bigtriangledown$ `Claim 1.1`

Claim 1.2: *$\partial_1\, g(p_1,p_2) \ge 0$ for all $(p_1,p_2) \in [0,1]^2$.*

Proof: $\partial_1\, g(p_1,p_2) \;=\; (8\,p_2 + 4\,p_2^2 + 4)\,p_1^3 + 10\,p_2 + 2 - 12\,p_2^2 + 3\,p_2^4$. Thus, $\partial_1\, g(p_1,p_2) < 0$ if and only if $-p_1^3 \;>\; h(p_2)$, where

$$h(p_2) \quad := \quad \frac{2 + 10\,p_2 - 12\,p_2^2 + 3\,p_2^4}{8\,p_2 + 4\,p_2^2 + 4}.$$

The denominator of $h(p_2)$ is clearly positive for $p_2 \in [0,1]$. The numerator of $h(p_2)$ is $H(p_2) := 2 + 10\,p_2 - 12\,p_2^2 + 3\,p_2^4$. It suffices to show that $H(p_2) \ge 0$ for $p_2 \in [0,1]$. But $H'(p_2) \;=\; 10 - 24\,p_2 + 12\,p_2^3$ has only one root in $[0,1]$, which corresponds to a (positive) maximum of H. Thus, H has no interior minima in $[0,1]$. Now, $H(0) = 2 > 0$ and $H(1) = 3 > 0$; thus, $H(p_2) > 0$ for all $p_2 \in [0,1]$. Thus, $h(p_2) > 0$ for all $p_2 \in [0,1]$, so it is impossible for $-p_1^3 \;>\; h(p_2)$ (because $p_1 > 0$). Thus, $\partial_1\, g(p_1,p_2) \ge 0$. $\bigtriangledown$ `Claim 1.2`

Claim 1.3: *$g(p_1,p_2) \ge 0$ for all $(p_1,p_2) \in [0,1]^2$.*

Proof: Claim 1.2 implies that $g(p_1,p_2)$ is increasing in p_1; thus, it suffices to check that $g(0,p_2) \ge 0$ for all $p_2 \in [0,1]$. But $g(0,p_2) = G(p_2) := 8\,p_2 - 12\,p_2^2 + 4\,p_2^3$. Now, $G'(p_2) = 8 - 24\,p_2 + 12\,p_2^2$ has roots $1 \pm \sqrt{3}/3$. Only one of these roots is in $[0,1]$, and it corresponds to a maximum of G. Also, $G(0) = 0 = G(1)$. Thus, $G(p_2) \ge 0$ for all $p_2 \in [0,1]$. Thus, $g(p_1,p_2) \ge 0$ for all $(p_1,p_2) \in [0,1]^2$. $\bigtriangledown$ `Claim 1.3`

Claims 1.1 and 1.3 together imply that $f(p_1,p_2) \ge 0$ for all $(p_1,p_2) \in [0,1]^2$. $\diamondsuit$ `Claim 1`

Claim 1 and Corollary E(a) imply that the $\overline{\Pi}$-maximizing contract is tenure-track. It remains to demonstrate the declining effort profile. The maximum of $\hat{\Pi}$ occurs along the boundary $p_2 = 1$. Thus, to identify p_1^*, it suffices to maximize

$$\Upsilon(p_1) \quad := \quad \frac{\hat{\Pi}(p_1,1)}{k^2} \;\underset{\text{(D.1)}}{=\!=}\; \frac{p\,(16\,p - 16\,p^2 + 4\,p^3 + 1)}{16(1 + 2\,p)}$$

The zeros of

$$\Upsilon'(p_1) \quad = \quad \frac{32\,p_1 + 1 + 24\,p_1^4 - 48\,p_1^3 - 16\,p_1^2}{16\,(1 + 2\,p_1)^2}$$

are the zeros of the numerator $32\,p_1+1+24\,p_1^4-48\,p_1^3-16\,p_1^2$. Only one of these zeros is in the interval $[0,1]$; it is located at $p_1^*\approx 0.8422568359$, and corresponds to a maximum of Υ. Since $p_1^*>1/2$, Corollary E(b) implies that $e_1^*>e_2^*$. □

Lemma G *Suppose* $\{\rho_e\}_{e\in\mathbb{R}_{\neq}}$ *is the family of exponential distributions from* Theorem 1(b). *Then the* $\overline{\Pi}$*-maximizing MNQ contract is tenure-track, with a declining effort profile.*

Proof: We have $P(e,s)=\exp(-s/e)$, so $\varsigma(e,p)=-e\,\ln(p)$. Also, $\overline{R}(e,s)=e+s$. Setting $S(p)=-\ln(p)$ and $c_1=c_2=1$, we see that $\{\rho_e\}_{e\in\mathbb{R}_{\neq}}$ is tenable; thus, we can apply Corollary E. In Lemma D, we have $L(p)=(1-\ln(p))$. Substitute into (D.3) to get

$$\begin{aligned}\Xi(p_1,p_2) &= \lambda(p_1,p_2)+p_1 g(p_1,p_2), \qquad\qquad (\text{G.1})\\ \text{where}\qquad g(p_1,p_2) &:= 2-p_2^2+p_1+2\,p_2+2\,p_1p_2+p_1p_2^2,\end{aligned}$$

and

$$\begin{aligned}\lambda(p_1,p_2) \;:=\;& \left(2\,p_1+p_1^2+p_1^2p_2^2+2\,p_1p_2+2\,p_1^2p_2\right)\ln{(p_1)}^2 \;+\; \left(p_1p_2^2+2\,p_1p_2+2\,p_2\right)\ln{(p_2)}^2\\ &-\;\left(4\,p_1+4\,p_1p_2+4\,p_1^2p_2+2\,p_1^2+2\,p_1^2p_2^2\right)\ln{(p_1)} \;-\; \left(2\,p_1p_2+2\,p_2\right)\ln{(p_2)}\,.\end{aligned}$$

Claim 1: $\Xi(p_1,p_2)>0$ *for all* $(p_1,p_2)\in[0,1]^2$.

Proof: $\lambda(p_1,p_2)\geq 0$ for all $(p_1,p_2)\in[0,1]^2$, because $\ln(x)^2\geq 0$ for all $x>0$, and $-\ln(x)\geq 0$ for all $x\in(0,1]$. Thus, it suffices to show $g(p_1,p_2)>0$. Let $h(p_2):=-p_2^2+2p_2+2$.

Claim 1.1: $g(p_1,p_2)>h(p_2)$ *for all* $p_1,p_2>0$.

Proof: Write $g(p_1,p_2)$ as polynomial in p_2 to get: $g(p_1,p_2)=(-1+p_1)\,p_2^2+(2+2\,p_1)\,p_2+2+p_1$. If $p_1>0$, then $-1+p_1>-1$, $2+2p_1>2$ and $2+p_1>2$. Thus, each p_2-coefficient of $g(p_1,p_2)$ is strictly larger than the corresponding coefficient of $h(p_2)$, for any $p_1>0$. Thus, $g(p_1,p_2)>h(p_2)$ for all $p_1,p_2>0$. ▽ `Claim 1.1`

Now, $h(0)=2>0$, $h(1)=3>0$, and h has no extremal points in $[0,1]$; thus $h(p_2)>0$ for all $p_2\in[0,1]$. Thus, Claim 2.1 implies that $g(p_1,p_2)>0$ for all $(p_1,p_2)\in[0,1]^2$. Thus, eqn.(G.1) implies that $\Xi(p_1,p_2)\geq 0$ for all $(p_1,p_2)\in[0,1]^2$, as desired. ◇ `Claim 1`

Claim 2 and Corollary E(a) imply that the $\overline{\Pi}$-maximizing contract is tenure-track. It remains to demonstrate the declining effort profile. The maximum of $\hat{\Pi}$ occurs along the boundary $p_2=1$. Thus, to identify p_1^*, it suffices to maximize

$$\Upsilon(p_1) \quad:=\quad \frac{\hat{\Pi}(p_1,1)}{k^2} \quad \underset{\text{(D.1)}}{=\!=} \quad \frac{p_1\left(4\,p_1-8\,p_1\ln{(p_1)}+4\,p_1\ln{(p_1)}^2+1\right)}{4(1+2\,p_1)}.$$

The zeros of

$$\Upsilon'(p_1) \quad=\quad \frac{-8\,p_1\ln{(p_1)}+8\,p_1\ln{(p_1)}^2+8\,p_1^2\ln{(p_1)}^2+1-8\,p_1^2}{4\,(1+2\,p_1)^2}$$

are the zeros of the numerator $-8\,p_1 \ln(p_1) + 8\,p_1 \ln(p_1)^2 + 8\,p_1^2 \ln(p_1)^2 + 1 - 8\,p_1^2$. This is a transcendental function, and it is not possible to find closed-form expressions for its zeros. However, numerically, the numerator has only one zero, located at $p_1^* \approx 0.7121849555$; this corresponds to the unique maximum of $\Upsilon(p_1)$. Since $p_1^* > 1/2$, Corollary E(b) implies that $e_1^* > e_2^*$. □

Lemma H *For any $\alpha > 1$, let $\{\rho_e^\alpha\}_{e\in\mathbb{R}_{\not-}}$ be the family of power law distributions from* Theorem 1(c)*. There exist $\underline{\alpha}, \overline{\alpha} \in (1,\infty)$ such that, if $\alpha \in (1,\underline{\alpha})$ or $\alpha \in (\overline{\alpha},\infty)$, then the $\overline{\Pi}$-maximizing MNQ contract is tenure-track, with a declining effort profile. In particular, this holds if $\alpha = 2$.*

Proof: For any $\alpha > 1$, we have $P_\alpha(e,s) = \left(\frac{e}{e+s}\right)^\alpha$; thus $\varsigma_\alpha(e,p) = e(p^{-1/\alpha} - 1)$. Also, $\overline{R}_\alpha(e,s) = \frac{e+\alpha s}{\alpha-1}$. Thus, setting $S_\alpha(p) := (p^{-1/\alpha} - 1)$, $c_1 = 1/(\alpha-1)$ and $c_2 = \alpha/(\alpha-1)$, we see that $\{\rho_e^\alpha\}_{e\in\mathbb{R}_{\not-}}$ is tenable. In Lemma D, we have

$$L_\alpha(p) \quad = \quad \frac{\alpha p^{-1/\alpha}}{(\alpha-1)} - 1 \quad \text{thus} \quad L'_\alpha(p) \quad = \quad \frac{-1}{(\alpha-1)\,p^{\frac{\alpha+1}{\alpha}}}.$$

Substituting into eqn.(D.3) we get $\Xi_\alpha(p_1,p_2) = \xi(p_1,p_2)/(\alpha-1)^2$, where $\xi(p_1,p_2) :=$ $2\,p_1 + 2\,p_2 - 4\,pa - 4\,p_2 a - 2\,p_1 p_2^{\frac{\alpha-2}{\alpha}}\alpha + p_1^2 + 4\,p_1^{\frac{2\alpha-1}{\alpha}} a p_2 + 2\,p_1^{\frac{2\alpha-1}{\alpha}} p_2^2\alpha + 4\,p_1 p_2 + p_1 p_2^{2\frac{\alpha-1}{\alpha}}\alpha^2 -$ $2\,p_2^{\frac{\alpha-2}{\alpha}}\alpha - 4\,p_2^{\frac{\alpha-1}{\alpha}}\alpha^2 + p_1 p_2^2 + 2\,p_1 p_2^{\frac{\alpha-2}{\alpha}}\alpha^2 + 6\,p_1 p_2^{\frac{\alpha-1}{\alpha}}\alpha + 2\,p_1^{\frac{\alpha-2}{\alpha}}\alpha^2 + 2\,p_1^{\frac{\alpha-2}{\alpha}}\alpha^2 p_2 + p_1^2 p_2^2 + 4\,p_1^{\frac{\alpha-1}{\alpha}}\alpha -$ $2\,p_1^{\frac{2\alpha-1}{\alpha}}\alpha^2 + 2\,\alpha^2 p_2 + p_1^{2\frac{\alpha-1}{\alpha}} p_2^2\alpha^2 + 4\,p_1\alpha^2 p_2 - 8\,p_1 p_2 a + 2\,p_1\alpha^2 - 2\,p_1 p_2^{\frac{\alpha-1}{\alpha}} - 4\,p_1^{\frac{\alpha-1}{\alpha}}\alpha^2 p_2 -$ $4\,p_1^{\frac{2\alpha-1}{\alpha}}\alpha^2 p_2 + p_1^{2\frac{\alpha-1}{\alpha}}\alpha^2 + 2\,p_1^{\frac{2\alpha-1}{\alpha}}\alpha - 2\,p_1 p_2^{\frac{2\alpha-1}{\alpha}} - 2\,p_2^{\frac{\alpha-1}{\alpha}} + 2\,p_2^{\frac{\alpha-2}{\alpha}}\alpha^2 + 6\,p_2^{\frac{\alpha-1}{\alpha}}\alpha + 4\,p_1 p_2^{\frac{2\alpha-1}{\alpha}}\alpha +$ $2\,p_1^2 p_2 + p_1\alpha^2 p_2^2 - 2\,pap_2^2 - 2\,p_1^{\frac{2\alpha-1}{\alpha}} p_2^2\alpha^2 + 2\,p_1^{2\frac{\alpha-1}{\alpha}}\alpha^2 p_2 - 2\,p_1 p_2^{\frac{2\alpha-1}{\alpha}}\alpha^2 - 2\,p_1 p_2^{2\frac{\alpha-1}{\alpha}}\alpha - 2\,p_1^2\alpha +$ $2\,p_1^2\alpha^2 p_2 - 4\,p_1^2 p_2 a + p_1^2 p_2^2\alpha^2 - 2\,p_1^2 p_2^2\alpha + p_1^2\alpha^2 - 4\,p_1 p_2^{\frac{\alpha-1}{\alpha}}\alpha^2 + 4\,p_1^{\frac{\alpha-1}{\alpha}} a p_2 - 4\,p_1^{\frac{\alpha-1}{\alpha}}\alpha^2$. Thus, it suffices to show that $\xi_\alpha(p_1,p_2) > 0$ for all $(p_1,p_2) \in [0,1]^2$. As promised in Theorem 1(b), we consider only the asymptotics as $\alpha \searrow 1$ or $\alpha \to \infty$.

Asymptotics as $\alpha \searrow 1$. We have

$$\lim_{\alpha\searrow 1} \xi_\alpha(p_1,p_2) \quad = \quad \xi_1(p_1,p_2) \quad := \quad (1-p_1) + 2\,p_2 + p_2^2 + 2\,\frac{p_2}{p_1} + \frac{2}{p_1}. \tag{H.1}$$

Now, $\xi_1(p_1,p_2)$ is positive for all $(p_1,p_2) \in [0,1]^2$, because $(1-p_1) \geq 0$ if $p_1 \leq 1$, and all the other terms in expression (H.1) are nonegative. Thus, if α is small enough, then $\xi_\alpha(p_1,p_2) > 0$ for all $(p_1,p_2) \in [0,1]^2$; hence Corollary E(a) implies that the $\overline{\Pi}$-maximizing contract is tenure-track.

Indeed, if $\alpha = 2$, we have $\xi_2(p_1,p_2) = 12 - 6\,\sqrt{p_2} + 10\,p_2 - 8\,p_1^{3/2} p_2 - 2\,p_1\,p_2^{3/2} - 8\,\sqrt{p_1}p_2 - 4\,p_1^{3/2}p_2^2 - 4\,p_1^{3/2} - 8\,\sqrt{p_1} - 6\,p_1\,\sqrt{p_2} + 12\,p_1\,p_2 + 5\,p_1\,p_2^2 + p_1^2 p_2^2 + p_1^2 + 2\,p_1^2 p_2 + 10\,p_1$. A numerical plot reveals that $6 < \xi_2(p_1,p_2) < 16$ for all $(p_1,p_2) \in [0,1]^2$. Thus, the $\overline{\Pi}$-maximizing contract is tenure-track when $\alpha = 2$.

It remains to demonstrate the declining effort profile. For all $p \in [0,1]$, we have $\lim_{\alpha \searrow 1} S_\alpha(p) - \dfrac{c_1(1-2p)}{2c_2 p} = \dfrac{1}{2p} > 0$. Thus, if α is small enough, then Corollary E(b) implies that $e_1^* > e_2^*$, as desired.

Asymptotics as $\alpha \to \infty$. A computation reveals that $\lim_{\alpha\to\infty} \xi_\alpha(p_1,p_2) = \Xi(p_1,p_2)$, where $\Xi_\alpha(p_1,p_2)$ is exactly as in eqn.(G.1) from the exponential case. Thus, Claim 2 implies that $\lim_{\alpha\to\infty} \xi_\alpha(p_1,p_2) > 0$ for all $(p_1,p_2) \in [0,1]^2$. Thus, if α is sufficiently large, then Corollary E(a) implies that the $\overline{\Pi}$-maximizing contract is tenure-track.

It remains to demonstrate the declining effort profile. Substituting $L_\alpha(p) = \frac{\alpha p^{-1/\alpha}}{(\alpha-1)} - 1$ into eqn.(D.1) and differentiating yields

$$\partial_1 \hat{\Pi}_\alpha(p_1,1) \quad = \quad k^2\, \frac{8\alpha^2\, f_\alpha(p_1) \; + \; \alpha\, g_\alpha(p_1) \; + \; h_\alpha(p_1)}{4(\alpha-1)^2(1+2\,p_1)^2}, \tag{H.2}$$

where $f_\alpha(p) := \left(p^2 + p^{2-\frac{2}{a}} - 2p^{2-\frac{1}{a}}\right) + \left(p + p^{1-\frac{2}{a}} - 2p^{1-\frac{1}{a}}\right)$, $g_\alpha(p) := -16\,p^{2\,\frac{a-1}{a}} - 8\,p^{\frac{a-2}{a}} + 32\,p^{\frac{2\,a-1}{a}} + 24\,p^{\frac{a-1}{a}} - 16\,p^2 - 16\,p$, and $h_\alpha(p) := 1 + 8\,p - 16\,p^{\frac{2\,a-1}{a}} - 8\,p^{\frac{a-1}{a}} + 8\,p^2$.

Claim 1: *If $p \in (0,1)$, then $f_\alpha(p) > 0$.*

Proof: If $p \in (0,1)$ then the function $x \mapsto p^x$ is convex. Thus, $p^x + p^y > 2p^{(x+y)/2}$. Setting $x = 2$ and $y = 2 - \frac{2}{\alpha}$, we get $p^2 + p^{2-\frac{2}{a}} > 2p^{2-\frac{1}{a}}$. Setting $x = 1$ and $y = 1 - \frac{2}{\alpha}$, we get $p^1 + p^{1-\frac{2}{a}} > 2p^{1-\frac{1}{a}}$. Thus, each of the two bracketed terms in $f_\alpha(p)$ is strictly positive; thus $f_\alpha(p) > 0$, as desired. ◇ `Claim 1`

In the limit as $\alpha\to\infty$, the term $(\alpha-1)^2$ in the denominator of expression (H.2) annihilates all terms in the numerator except $f_\alpha(p)$. Thus, if α is extremely large, then the sign of $\partial_1\hat{\Pi}_\alpha(p_1,1)$ is the same as the sign of $f_\alpha(p_1)$, and $f_\alpha(p_1) > 0$ by Claim 3. Thus, $\partial_1\hat{\Pi}_\alpha(p_1,1) > 0$ for all $p \in (0,1)$; thus, the optimal value of p_1 is $p_1^* = 1$.

This means that, if α is large enough, then $p_1^* > 1/2$; thus, Corollary E(b) implies that $e_1^* > e_2^*$, as desired. □

Proof of Theorem 1. Lemmas F, G, and H state that, under any of the hypotheses (a), (b) or (c), the $\overline{\Pi}$-maximizing element in the space of MNQ contract is tenure-track, and induces a declining profile of effort. Thus, by Proposition 4, the same statement is true for the $\overline{\Pi}$-maximizing element in the space of *all* contracts. Thus, by Proposition 3, the same statement is true for the $\overline{\Pi}$-maximizing element in the space of *raid-proof* contracts. This proves Theorem 1. □

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